

Computation of the $O(p^6)$ order low-energy constants: an update

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We update our original low-energy constants to the $O(p^6)$ order, including two and three flavours, the normal and anomalous ones. Following a comparative analysis, the $O(p^4)$ results are considered better. In the $O(p^6)$ order, most of our results are consistent or better with those we have found in the literature, although several are worse.

PACS numbers: 12.39.Fe, 11.30.Rd, 12.38.Aw, 12.38.Lg

I. INTRODUCTION

Chiral perturbation theory (ChPT) is a useful method to solve low-energy mesons interactions. A significant problem is to establish the chiral Lagrangian (CL). Until now, the monomials of CL have been obtained to the $O(p^6)$ order [1–11], including both the normal and anomalous parts, two and three flavours, and the special unitary and unitary groups. The $O(p^6)$ order monomials seem accurate enough to describe the present experiments. In contrast, the coefficients for each of the monomials, called the low-energy constants (LECs), also play an important role of ChPT. Concrete numerical results need them. However ChPT does not provide these LECs, and hence they need to be obtained from other theories. Because of non-perturbative effects, exact values for the LECs are hard to obtain, especially at the $O(p^6)$ order. At the $O(p^4)$ order, although exact results have not been obtained, values from different methods are close. Their signs and magnitudes are the same. At the $O(p^6)$ order, there are various methods to obtain LECs, the more common ones being resonance chiral theory [12, 13], sum rules [14], lattice QCD [15, 16] holographic theory [17], QCD [18], and global fit [19]. Of course, LECs can also be extracted from experimental data. Some methods can obtain a value for a single LEC; while others produce values for combinations of LECs. Each method has its advantages and disadvantages, but some values do come with large errors because of non-perturbative QCD processes.

Several years ago, values of a few LECs obtained by different methods were scattered in various references. A large number of LECs remained unknown in value. To taking advantage of ChPT, LECs are needed for numerical calculations. Our motivation was to explore a method to obtain all the LECs in a single calculation. We hoped the method would be model-independent, as had been developed from the underlying QCD theory. Nevertheless at that time, the theoretical analysis was poor, and we only used simple methods that involved some rough approximations [18, 20]. Although expedient, preliminary results at least were obtained.

Given that the approximations are too rough, the results did yield a gauge-invariant, nonlocal, dynamical-quark (GND model) model [21]. Ref. [19] details checks of our $O(p^6)$ LECs (C_i) via a global fit of the $O(p^4)$ LECs (L_i) to the $O(p^6)$ order. The χ^2 divided by the degrees of freedom (χ^2/dof) is 4.13/4. If all LECs are multiplied by 0.27, $\chi^2/\text{dof}=1.20/3$ and hence the C_i values appeared to be too large. In statistical theory, when dof is large enough (a typical choice is $\text{dof}>30$), $\chi^2/\text{dof}\sim 1$. However at only 4, the dof is too small and it is not sufficiently large to assess the reliability of the calculation by χ^2/dof . Because of the χ^2 distribution, χ^2/dof is also not as small as it should be. Furthermore the $O(p^4)$ and $O(p^6)$ LECs are independent. Evaluating the confidence level of the C_i by L_i is not suitable, especially when dof is small. A more believable assessment can be garnered by comparing experiment data. However the absolute values of our L_i are too large, and may include some systematic errors in the calculation, such as those associated with the large N_C limit. These systematic errors must exist at all orders. With the passing of

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time, it is now the moment improve the $O(p^6)$ LECs. While tedious, these needs to be improved step by step. This paper research analyses the origin of the systematic errors and tries to remedy them as precisely as possible.

This paper is organized as follows: In Sec. II, we review our method to obtain the CL from QCD and introduce a more precise approximation. In Sec. III, a concrete method to calculate LECs is introduced. In Sec. IV, we list our results for the LECs, both normal and anomalous, up to and including order $O(p^6)$. In Sec. V, we compare our results with others in the literature, and check for new predictions. Section VI concludes with a summary.

II. REVIEW AND IMPROVEMENTS OVER PREVIOUS CALCULATIONS OF LECs

In a previous work, we obtained the action of the chiral theory in the large N_c limit. Drived from first principles QCD, it takes the form [22, 23]

$$S_{\text{eff}} = -iN_c \text{Tr} \ln[i\partial + J_\Omega - \Pi_{\Omega c}] + N_c \int d^4x d^4x' \Phi_{\Omega c}^{\sigma\rho}(x, x') \Pi_{\Omega c}^{\sigma\rho}(x, x') + N_c \sum_{n=2}^{\infty} \int d^4x_1 \cdots d^4x'_n \\ \times \frac{(-i)^n (N_c g_s^2)^{n-1}}{n!} \bar{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x'_1, \cdots, x_n, x'_n) \Phi_{\Omega c}^{\sigma_1 \rho_1}(x_1, x'_1) \cdots \Phi_{\Omega c}^{\sigma_n \rho_n}(x_n, x'_n) \\ + iN_c \int d^4x \text{tr}_{lf} \{ \Xi_c(x) [-i \sin \frac{\vartheta_c(x)}{N_f} + \gamma_5 \cos \frac{\vartheta_c(x)}{N_f}] \Phi_{\Omega, c}^T(x, x) \}, \quad (1)$$

in which J_Ω is the external source J including currents and densities after making a Goldstone-field-dependent chiral rotation Ω :

$$J_\Omega = [\Omega P_R + \Omega^\dagger P_L][J + i\partial][\Omega P_R + \Omega^\dagger P_L] = \not{J}_\Omega + \not{J}_\Omega \gamma_5 - s_\Omega + ip_\Omega \gamma_5, \quad J = \not{J} + \not{J} \gamma_5 - s + ip\gamma_5, \quad U = \Omega^2. \quad (2)$$

where $\Phi_{\Omega c}$ and $\Pi_{\Omega c}$ are, respectively, the two-point rotated-quark Green's functions and the interaction part of two-point rotated-quark vertices in the presence of external sources; $\Phi_{\Omega c}$ is defined by

$$\Phi_{\Omega c}^{\sigma\rho}(x, y) \equiv \frac{1}{N_c} \langle \bar{\psi}_\Omega^\sigma(x) \psi_\Omega^\rho(y) \rangle = -i[(i\partial + J_\Omega - \Pi_{\Omega c})^{-1}]^{\sigma\rho}(y, x), \quad \psi_\Omega(x) \equiv [\Omega(x)P_L + \Omega^\dagger(x)P_R]\psi(x), \quad (3)$$

$$\Pi_{\Omega c}^{\sigma\rho}(x, y) = -\tilde{\Xi}^{\sigma\rho}(x) \delta^4(x - y) - \sum_{n=1}^{\infty} \int d^4x_1 \cdots d^4x_n d^4x'_1 \cdots d^4x'_n \frac{(-i)^{n+1} (N_c g_s^2)^n}{n!} \\ \times \bar{G}_{\rho\rho_1 \cdots \rho_n}^{\sigma\sigma_1 \cdots \sigma_n}(x, y, x_1, x'_1, \cdots, x_n, x'_n) \Phi_{\Omega c}^{\sigma_1 \rho_1}(x_1, x'_1) \cdots \Phi_{\Omega c}^{\sigma_n \rho_n}(x_n, x'_n), \quad (4)$$

with subscript c denoting the classical field and $\psi(x)$ the light quark fields. $\bar{G}_{\rho_1 \cdots \rho_n}^{\sigma_1 \cdots \sigma_n}(x_1, x'_1, \cdots, x_n, x'_n)$ is the effective gluon n -point Green's function and g_s is the coupling constant of QCD. The $\Phi_{\Omega c}$ and $\Pi_{\Omega c}$ are related by the first equation of (3) and determined by

$$[\Phi_{\Omega c} + \tilde{\Xi}]^{\sigma\rho} + \sum_{n=1}^{\infty} \int d^4x_1 d^4x'_1 \cdots d^4x_n d^4x'_n \frac{(-i)^{n+1} (N_c g_s^2)^n}{n!} \bar{G}_{\rho\rho_1 \cdots \rho_n}^{\sigma\sigma_1 \cdots \sigma_n}(x, y, x_1, x'_1, \cdots, x_n, x'_n) \\ \times \Phi_{\Omega c}^{\sigma_1 \rho_1}(x_1, x'_1) \cdots \Phi_{\Omega c}^{\sigma_n \rho_n}(x_n, x'_n) = O(\frac{1}{N_c}), \quad (5)$$

where ϑ_c is the phase angle of the $U(1)$ factor, and Ξ_c and $\tilde{\Xi}$ are two parameters in the calculation, defined by Eqs. (21) and (65) in Ref.[22]. In this work, they have little importance and are neglected. Eq. (5) is the Schwinger-Dyson equation (SDE) in the presence of the rotated external source. In Ref.[23], we have assumed an approximate solution of (5) given by

$$\Pi_{\Omega c}^{\sigma\rho}(x, y) = [\Sigma(\bar{\nabla}_x^2)]^{\sigma\rho} \delta^4(x - y), \quad \bar{\nabla}_x^\mu = \partial_x^\mu - iv_\Omega^\mu(x), \quad (6)$$

where Σ is the quark self-energy which satisfies SDE (5) with vanishing rotated external source. Under the ladder approximation, this SDE in Euclidean space-time is reduced to the standard form

$$\Sigma(p^2) - 3C_2(R) \int \frac{d^4q}{4\pi^3} \frac{\alpha_s[(p-q)^2]}{(p-q)^2} \frac{\Sigma(q^2)}{q^2 + \Sigma^2(q^2)} = 0, \quad (7)$$

where $\alpha_s(p^2)$ is the running coupling constant of QCD which depends on N_C and the number of quark flavours, and $C_2(R)$ is the second-order Casimir operator of the quark representation R . In this work, the quarks belonging to the

$SU(N_C)$ fundamental representation, and therefore $C_2(R) = (N_c^2 - 1)/2N_c$; in the large N_C limit, the second term is neglected.

In our previous article, because of the computational complexity, we did not calculate all terms in Eq. (1), but introduced some approximations to reduced the last three terms and leave only the first. The approximations are rough, and the results reproduce those of the GND model [21]. The last three terms could have introduced some systematic errors. The absolute values of the previous results seem too large when compared with others in the literature. The purpose of this paper is to remedy these errors partially and produce a more credible result.

For now, we have no idea how to solve the last term in (1) and hence omit it; we shall only focus on the second and third terms. The infinite sum in both the third terms in (1) and the second term in (4) appear similar but they are not equal. To find the source of the systematic errors and to maintain a manageable calculation we only retain the two-point contributions, the terms with $n = 2$ in (1) and $n = 1$ in (4). When we simplify SDE from (5) to (7), we also include $n = 1$ in (5). The present approximation and the approximation for SDE are all to first order, omitting all higher order contributions; they are then at the same level of accuracy. The other terms describe mesonic interactions, which we consider as less important when compared previous terms. The final results will substantiate this decision. Hence we shall add an additional effective action,

$$\Delta S_{\text{eff}} \sim \frac{1}{2}N_c \int d^4x d^4x' \Phi_{\Omega_c}^{\sigma\rho}(x, x') \Pi_{\Omega_c}^{\sigma\rho}(x, x') \quad (8)$$

$$\sim -\frac{i}{2}N_c \text{Tr}[(i\not{\partial} + J_\Omega - \Sigma(\bar{\nabla}^2))^{-1} \Sigma(\bar{\nabla}^2)]. \quad (9)$$

the approximation of which has been used in (6).

With the same considerations for the anomalous parts, we only calculated the first term in (1), although using a different method to introduce the fifth dimensional integral [20]. We need not repeat this as the additional effective Lagrangian is the same as (9). Therefore, for the present study, we calculate (9) including both the normal and anomalous parts.

III. CALCULATION OF THE ADDITIONAL TERMS

To calculate the additional terms, we first use the Wick rotation to change (9) to Euclidean spacetime as [18, 20], and then expand it as a Taylor series,

$$\Delta S_{\text{eff}} = -\frac{1}{2}N_c \text{Tr}[(D + \Sigma(-\bar{\nabla}^2))^{-1} \Sigma(-\bar{\nabla}^2)] \quad (10)$$

$$= -\frac{1}{2}N_c \text{tr}[(-i\not{k} + D + \Sigma((k + i\bar{\nabla})^2))^{-1} \Sigma((k + i\bar{\nabla})^2)] \quad (11)$$

$$= -\frac{1}{2}N_c \int d^4x \int \frac{d^4k}{(2\pi)^4} \text{tr}[(\Sigma_k X + i\not{k} X) \sum_{n=0}^{\infty} (-1)^n [(D + \Sigma_1)(\Sigma_k X + i\not{k} X)]^n \Sigma((k + i\bar{\nabla})^2)], \quad (12)$$

where “Tr” includes the traces over coordinate space, spinor space and flavour space, “tr” includes only the traces over spinor and flavour space, $D \equiv \not{\nabla} - i\not{q}_\Omega \gamma_5 - s_\Omega + ip_\Omega \gamma_5$, and $\Sigma_1 \equiv \Sigma((k + i\bar{\nabla})^2) - \Sigma(k^2)$.

To a given order ($O(p^2)$, $O(p^4)$ or $O(p^6)$), after the tracing over the spinor space, the series are

$$\Delta S_{\text{eff}}^{2n} = \int d^4x \sum_{k=1}^m a_k \langle O_{\Omega,k} \rangle \quad (13)$$

where a_k are coefficients, $O_{\Omega,k}$ are monomials with flavour indices including $\bar{\nabla}^\mu$, a_Ω^μ , s_Ω , p_Ω , and “ $\langle \dots \rangle$ ” means trace over flavour space. In (13), we have used the basic relations to simplify the results, including trace relations, the Einstein summation convention, and for the anomalous terms also including the Schouten identity. Nevertheless, not all of the $O_{\Omega,k}$ are independent.

Generally, the number of $O_{\Omega,k}$ is larger than the number of linear independent terms O_l . Specifically, the relationship between the two is given by

$$\langle O_l \rangle = \sum_{k=1}^m A_{lk} \langle O_{\Omega,k} \rangle, \quad l = 1, 2, 3, \dots, M \quad (14)$$

where M is the number of LECs up to a given order.¹ This implies that generally A_{lk} is not a square matrix. The reduced row echelon form of A_{lk} is

$$A_{lk} \rightarrow B_{lk} = \begin{pmatrix} 1 & C_{12} & O & C_{14} & O & \cdots & \cdots & \cdots \\ & & 1 & C_{24} & O & \cdots & \cdots & \cdots \\ & & & \cdots & \cdots & \cdots & \cdots & \cdots \\ & & & & 1 & \cdots & \cdots & \cdots \end{pmatrix}, \quad (15)$$

where the bottom-left corner contains just zero elements with O representing a zero matrix of the appropriate dimension C representing possibly a non-zero matrix and “ \cdots ” also nonzero matrices. The rank of A_{lk} or B_{lk} is equal to the number of independent linear bases with each nonzero row-vector in B_{lk} corresponding to a linear basis in $\langle O_\Omega \rangle$. We select those $O_{\Omega,k'}$ that are independent, and set $B_{k'k'} = 1$ and $B_{k'k'}$ to be the first non-zero elements in the k' th row in B_{lk} . All dependent $O_{\Omega,k}$ can be replaced by $O_{\Omega,k'}$.

Without using the Cayley-Hamilton relations, the LECs K_l for arbitrary N_f flavours are defined as

$$S_{\text{eff}}^{2n} = \int d^4x \sum_{l=1}^M K_l \langle O_l \rangle = \int d^4x \sum_{l=1}^M \sum_{k=1}^m K_l A_{lk} \langle O_{\Omega,k} \rangle. \quad (16)$$

In the second equation, we have used (14). Comparing (13) and (16), because all the relations in N_f flavours have been used the coefficients in front of $\langle O_{\Omega,k} \rangle$ need to be equal.

$$a_k = \sum_{l=1}^M K_l A_{lk}, \quad k = 1, 2, 3, \dots, m. \quad (17)$$

In (17), there are m equations in M variables with $m > M$. We select the M independent $O_{\Omega,k'}$ to solve the equations, leaving the remaining $m - M$ equations as constraints. Hence the additional LECs are

$$\Delta K_l = \sum_{k'} a_{k'} A_{k'l}^{-1}. \quad (18)$$

Replacing ΔK_l in (17), the second equal sign must hold. These are the constraint equations.

Finally, using the Cayley-Hamilton relations, all the LECs can be obtained for three and two flavours.

IV. RESULTS

A. Order $O(p^2)$ and $O(p^4)$

To the $O(p^2)$ order, the additional analytical LECs are

$$\begin{aligned} \Delta F_0^2 &= \int \frac{d^4k}{(2\pi)^4} [4\Sigma_k^2 X^2 - 6\Sigma_k^4 X^3], \\ \Delta F_0^2 B_0 &= \int \frac{d^4k}{(2\pi)^4} [2\Sigma_k X - 4\Sigma_k^3 X^2], \\ X &\equiv \frac{1}{k^2 + \Sigma_k^2}, \quad \Sigma_k \equiv \Sigma(k^2). \end{aligned} \quad (19)$$

The analytical results in (19) and the higher order in the following are all set in Euclidean spacetime. A more precise result, with all terms on the right hand side of (1), has been given in [22]. This is just the Pagels-Stokar formula for F_0^2 and also the $\Lambda \rightarrow \infty$ results in our previous work [18]. It seems that only the first term in (1) is sufficient to give the $O(p^2)$ results, the other terms do not contribute at the $O(p^2)$ order. In considering the accuracy, we use the original results in [18] at the $O(p^2)$ order. The method described in [22], however, is not easily extendible to higher order, and there is no proof that indicates that the other terms in (1) make no contribution to the LECs. Hence at higher order, we consider (12).

¹ The building blocks of O_l and O_Ω are different. Their relations can be found in Table XV in Ref.[18]

At the $O(p^4)$ order, the additional analytical results are

$$\begin{aligned}
\Delta C_2 &= \int \frac{d^4 k}{(2\pi)^4} \left[-\Sigma_k'^2 X + 11\Sigma_k^2 \Sigma_k'^2 X^2 - 34\Sigma_k^4 \Sigma_k'^2 X^3 - 2\Sigma_k^4 X^4 + 40\Sigma_k^6 \Sigma_k'^2 X^4 + 4\Sigma_k^6 X^5 - 16\Sigma_k^8 \Sigma_k'^2 X^5 \right], \\
\Delta C_3 &= \int \frac{d^4 k}{(2\pi)^4} \left[-\frac{1}{2}\Sigma_k'^2 X + \frac{49}{6}\Sigma_k^2 \Sigma_k'^2 X^2 - \frac{2}{3}\Sigma_k^2 X^3 - \frac{83}{3}\Sigma_k^4 \Sigma_k'^2 X^3 - \frac{5}{3}\Sigma_k^4 X^4 + \frac{100}{3}\Sigma_k^6 \Sigma_k'^2 X^4 \right. \\
&\quad \left. + \frac{10}{3}\Sigma_k^6 X^5 - \frac{40}{3}\Sigma_k^8 \Sigma_k'^2 X^5 \right], \\
\Delta C_4 &= \int \frac{d^4 k}{(2\pi)^4} \left[-\frac{26}{3}\Sigma_k^2 X^3 + \frac{118}{3}\Sigma_k^4 X^4 - \frac{104}{3}\Sigma_k^6 X^5 \right], \\
\Delta C_5 &= \int \frac{d^4 k}{(2\pi)^4} \left[\frac{14}{3}\Sigma_k^2 X^3 - \frac{52}{3}\Sigma_k^4 X^4 + \frac{44}{3}\Sigma_k^6 X^5 \right], \\
\Delta C_6 &= \int \frac{d^4 k}{(2\pi)^4} \left[6\Sigma_k^2 X^2 - 8\Sigma_k^4 X^3 \right], \\
\Delta C_7 &= \int \frac{d^4 k}{(2\pi)^4} \left[2\Sigma_k^2 X^2 \right], \\
\Delta C_8 &= \int \frac{d^4 k}{(2\pi)^4} \left[-4\Sigma_k X^2 + 34\Sigma_k^3 X^3 - 36\Sigma_k^5 X^4 \right], \\
\Delta C_9 &= \int \frac{d^4 k}{(2\pi)^4} \left[\Sigma_k'^2 X - \frac{2}{3}\Sigma_k \Sigma_k' X^2 - 3\Sigma_k^2 \Sigma_k'^2 X^2 - \frac{2}{3}\Sigma_k^2 X^3 + \frac{2}{3}\Sigma_k^3 \Sigma_k' X^3 + 2\Sigma_k^4 \Sigma_k'^2 X^3 \right], \\
\Delta C_{10} &= \int \frac{d^4 k}{(2\pi)^4} \left[-2i\Sigma_k'^2 X + \frac{82i}{3}\Sigma_k^2 \Sigma_k'^2 X^2 + \frac{20i}{3}\Sigma_k^2 X^3 - \frac{268i}{3}\Sigma_k^4 \Sigma_k'^2 X^3 - \frac{52i}{3}\Sigma_k^4 X^4 + \frac{320i}{3}\Sigma_k^6 \Sigma_k'^2 X^4 \right. \\
&\quad \left. + \frac{32i}{3}\Sigma_k^6 X^5 - \frac{128i}{3}\Sigma_k^8 \Sigma_k'^2 X^5 \right], \\
\Delta C_{11} &= \int \frac{d^4 k}{(2\pi)^4} \left[-2\Sigma_k X^2 + 6\Sigma_k^3 X^3 \right]. \tag{20}
\end{aligned}$$

The definitions of C_i can be found in Eq. (19) of [21], and the relations between C_i and common L_i can be found in Eq. (24) in [21].

To obtain numerical results, we use the same quark self-energy Σ_k as in [18, 23] with the running coupling constant $\alpha_s(p^2)$ of [24]. To complete the integral, two other input parameters are needed. One is F_0 , for which we choose $F_0 = 87\text{MeV}$; the other is a cutoff Λ which comes from the calculation of the first term in (1). The details can be found in [18] and therefore we have chosen $\Lambda = 1.0^{+0.1}_{-0.1}\text{GeV}$. The three-flavour numerical results are listed in the second row in Table I. Those LECs that depend on Λ are expressible as

$$L_{\Lambda=1\text{GeV}} \left[\frac{L_{\Lambda=1.1\text{GeV}} - L_{\Lambda=1\text{GeV}}}{L_{\Lambda=0.9\text{GeV}} - L_{\Lambda=1\text{GeV}}} \right]. \tag{21}$$

The superscript and subscript indicate how the LECs are sensitive to Λ . For two flavours, we give the usual \bar{l}_i , $i = 1, 2, 3, 4, 5, 6$,

$$l_i = \frac{1}{32\pi^2} \gamma_i (\bar{l}_i + \ln \frac{M_\pi^2}{\mu^2}), \tag{22}$$

where the γ_i are given in Ref.[2]. These results are also listed in Table I.

Comparing the “new” and “old” results in Table I, the new absolute values of $L_i(\bar{l}_i)$ are, as expected, smaller than the older ones, except for \bar{l}_3 . For comparison, we also list the other results obtained by different methods: [2, 3] are the first results from experimental data; [25] gives the LECs from resonance chiral theory; [17] gives the LECs from a class of holographic theories; [19] gives the L_1, \dots, L_8 from the global fit of the $O(p^6)$ LECs, and L_9 and L_{10} are given in [26, 27], respectively. These are the usual methods to obtain LECs at present. On the whole, most of the old results are larger than the others, but the new results are closer. The only one new result larger than the old one is \bar{l}_3 , but is also much closer than the older one. Our new results are much closer to the experimental results, and most results are within the error uncertainties of the resonance results. Nevertheless they appear a bit far from the results from the global fit. One possible reason is that the global-fitted results do not maintain the large N_C limit, but L_4

TABLE I: The p^4 order LECs. Λ_{QCD} is in unit of GeV, and L_1, \dots, L_{10}, l_7 are in units 10^{-3} .

| $N_f = 3$ | Λ_{QCD} | L_1 | L_2 | L_3 | L_4 | L_5 | L_6 | L_7 | L_8 | L_9 | L_{10} |
|-----------|------------------------|-------------------------|------------------------|-------------------------|------------------------|-------------------------|-------------------------|-------------------------------------|------------------------|------------------------|-------------------------|
| New | 453_{-12}^{-6} | $0.92_{-0.04}^{+0.03}$ | $1.84_{-0.08}^{+0.05}$ | $-4.94_{+0.21}^{-0.14}$ | 0_{-0}^{+0} | $1.26_{-0.06}^{+0.01}$ | 0_{-0}^{+0} | $-0.42_{-0.05}^{+0.04}$ | $0.84_{+0.04}^{-0.05}$ | $6.53_{-0.37}^{+0.24}$ | $-5.43_{+0.44}^{-0.29}$ |
| Old [18] | 453_{+12}^{-6} | $1.23_{-0.04}^{+0.03}$ | $2.46_{-0.08}^{+0.05}$ | $-6.85_{+0.21}^{-0.15}$ | 0_{-0}^{+0} | $1.48_{-0.03}^{-0.01}$ | 0_{-0}^{+0} | $-0.51_{-0.06}^{+0.05}$ | $1.02_{+0.06}^{-0.06}$ | $8.86_{-0.37}^{+0.24}$ | $-7.40_{+0.44}^{-0.29}$ |
| Ref.[3]: | | 0.9 ± 0.3 | 1.7 ± 0.7 | -4.4 ± 2.5 | 0 ± 0.5 | 2.2 ± 0.5 | 0 ± 0.3 | -0.4 ± 0.15 | 1.1 ± 0.3 | 7.4 ± 0.7 | -6.0 ± 0.7 |
| Ref.[25]: | | 0.4 ± 0.3 | 1.4 ± 0.3 | -3.5 ± 1.1 | -0.3 ± 0.5 | 1.4 ± 0.5 | -0.2 ± 0.3 | -0.4 ± 0.2 | 0.9 ± 0.3 | 6.9 ± 0.7 | -5.5 ± 0.7 |
| Ref.[17]: | | 0.5 | 1.0 | -3.2 | | | | | | 6.3 | -6.3 |
| Ref.[19] | | 0.88 ± 0.09 | 0.61 ± 0.20 | -3.04 ± 0.43 | 0.75 ± 0.75 | 0.58 ± 0.13 | 0.29 ± 0.85 | -0.11 ± 0.15 | 0.18 ± 0.18 | 5.93 ± 4.3 | -4.06 ± 0.39 |
| $N_f = 2$ | Λ_{QCD} | \bar{L}_1 | \bar{L}_2 | \bar{L}_3 | \bar{L}_4 | \bar{L}_5 | \bar{L}_6 | l_7 | | | |
| New | 465_{+12}^{-6} | $-2.33_{+0.24}^{-0.17}$ | $6.85_{-0.14}^{+0.09}$ | $2.33_{-0.36}^{+0.28}$ | $4.24_{-0.04}^{+0.00}$ | $13.55_{-0.80}^{+0.53}$ | $15.60_{-0.67}^{+0.44}$ | $3.61_{+0.80}^{-0.55}$ | | | |
| Old [18] | 465_{+12}^{-6} | $-4.77_{+0.24}^{-0.17}$ | $8.01_{-0.14}^{+0.09}$ | $1.97_{-0.35}^{+0.29}$ | $4.34_{-0.02}^{-0.01}$ | $17.35_{-0.80}^{+0.53}$ | $19.98_{-0.67}^{+0.44}$ | $4.18_{+0.97}^{-0.65}$ ^a | | | |
| Ref.[2]: | | -2.3 ± 3.7 | 6.0 ± 1.3 | 2.9 ± 2.4 | 4.3 ± 0.9 | 13.9 ± 1.3 | 16.5 ± 1.1 | $O(5)$ | | | |

^aThere exists a mistake in [18] for l_7 , this is a correction.

and L_6 have fits that are not very small, the effect of which propagate throughout the calculation and decrease the values of other LECs. So far, our calculation remains valid only in the large N_C limit, and therefore they are not very closed to the global fit results.

These observations indicate that our approximation in (8) is reasonable. Although we only selected $n = 2$ in (1) and $n = 1$ in (4), the tendency is clear. The second and the third terms in (1) carry the systematic error in our original calculations. To date Table I gives the leading order corrections. Hence, we believe that when we extend the calculations to the $O(p^6)$ order, the results will be more credible.

B. Order $O(p^6)$

Because our method only applies in the large N_C limit, for simplicity, in the $O(p^6)$ order, we only need to calculate the large N_C limit terms. Without the equations of motion, in large N_C limit, the CL is

$$\mathcal{L}_6 = \sum_{n=1}^{68} \tilde{K}_n \tilde{Y}_n. \quad (23)$$

These \tilde{Y}_n and their relationship to Y_i defined in [5] are listed in Table II, \tilde{K}_n are some coefficients.

As for the p^2 and p^4 orders, expanding (12) to the p^6 order, with the method in Sec. III, treating \tilde{Y}_n as $\langle O_{\Omega, k} \rangle$, Y_n as $\langle O_l \rangle$ in (16), we can obtain the $\Delta \tilde{K}_i$ coefficients. We have listed the values in (A1) in Appendix A. The final $O(p^6)$ LECs are listed in Table III.

$$C_{\Lambda=1\text{GeV}} \begin{vmatrix} C_{\Lambda=1.1\text{GeV}} - C_{\Lambda=1\text{GeV}} \\ C_{\Lambda=0.9\text{GeV}} - C_{\Lambda=1\text{GeV}} \end{vmatrix}, \quad C_{\Lambda=1\text{GeV}} \begin{vmatrix} C_{\Lambda=1.1\text{GeV}} - C_{\Lambda=1\text{GeV}} \\ C_{\Lambda=0.9\text{GeV}} - C_{\Lambda=1\text{GeV}} \end{vmatrix}.$$

Because of the new relation given in Ref. [6], we remove c_{37} as previously. Unlike the $O(p^4)$ order, some absolute values of the new LECs are smaller than the old ones, such as C_1 and C_4 ; some are almost unchanged, such as C_3 and C_{12} ; some are larger than the old ones, such as C_{52} and C_{65} ; and some even change signs, such as C_{22} and C_{69} . These arise because of the choice of the independent terms in [5] and the complex relations in (17).

The calculations are too highly complicated. To avoid possible mistakes, the expansion in (12) and most of the other calculations are done by computer. To check the correctness of our results, we examined them in various ways. First, some terms in Table II have two parts, which we calculated separately. C , P , and Hermitian invariance constrains the two parts of the coefficient as being equal or with a sign difference. Our analytical results for the separate parts must give the same coefficients. Second, if we switch off the quark self-energy, all the LECs, except the contact terms', must vanish[18]. This places a strong restriction on our results. Third, because of the strict constraint conditions in (17), we have $109 - 68 = 41$ constraint conditions, with 109 being the number of O_Ω in (16). They also impose strong restrictions on our results. With all the above assessments, we are confident of the reliability of our numerical results for $O(p^6)$ LECs.

TABLE II: \tilde{Y}_n and their relations to Y_i .

| n | \tilde{Y}_n | relations | n | \tilde{Y}_i | relations |
|-----|---|---|-----|--|---------------|
| 1 | $\langle u^\mu u_\mu h^{\nu\lambda} h_{\nu\lambda} \rangle$ | Y_1 | 35 | $\langle i f_+^{\mu\nu} u^\lambda u_\mu u_\lambda u_\nu + i f_+^{\mu\nu} u_\mu u^\lambda u_\nu u_\lambda \rangle$ | Y_{68} |
| 2 | $\langle h^{\mu\nu} u^\lambda h_{\mu\nu} u_\lambda \rangle$ | Y_3 | 36 | $\langle u^\mu u_\mu f_+^{\nu\lambda} f_{+\nu\lambda} \rangle$ | Y_{71} |
| 3 | $\langle h^{\mu\nu} u^\lambda h_{\mu\lambda} u_\nu + h^{\mu\nu} u_\nu h_\mu^\lambda u_\lambda \rangle$ | Y_5 | 37 | $\langle f_+^{\mu\nu} u^\lambda f_{+\mu\nu} u_\lambda \rangle$ | |
| 4 | $\langle u^\mu u_\mu u^\nu \chi_+ u_\nu \rangle$ | Y_7 | 38 | $\langle f_+^{\mu\nu} f_{+\mu}^\lambda u_\nu u_\lambda \rangle$ | Y_{75} |
| 5 | $\langle u^\mu u_\mu u^\nu \chi_+ u_\nu \rangle$ | Y_{11} | 39 | $\langle f_+^{\mu\nu} f_{+\mu}^\lambda u_\lambda u_\nu \rangle$ | Y_{76} |
| 6 | $\langle \chi_+ u^\mu u^\nu u_\mu u_\nu \rangle$ | Y_{13} | 40 | $\langle f_+^{\mu\nu} u^\lambda f_{+\mu\lambda} u_\nu + f_+^{\mu\nu} u_\nu f_{+\mu}^\lambda u_\lambda \rangle$ | Y_{78} |
| 7 | $\langle \chi_+ h^{\mu\nu} h_{\mu\nu} \rangle$ | Y_{17} | 41 | $\langle \chi_+ f_+^{\mu\nu} f_{+\mu\nu} \rangle$ | Y_{81} |
| 8 | $\langle u^\mu u_\mu \chi_+ \chi_+ \rangle$ | Y_{19} | 42 | $\langle i f_+^{\mu\nu} \chi_+ u_\mu u_\nu + i f_+^{\mu\nu} u_\mu u_\nu \chi_+ \rangle$ | Y_{83} |
| 9 | $\langle \chi_+ u^\mu \chi_+ u_\mu \rangle$ | Y_{23} | 43 | $\langle i f_+^{\mu\nu} u_\mu \chi_+ u_\nu \rangle$ | Y_{85} |
| 10 | $\langle \chi_+ \chi_+ \chi_+ \rangle$ | Y_{25} | 44 | $\langle f_-^{\mu\nu} h_\nu^\lambda u_\lambda u_\mu + f_-^{\mu\nu} u_\mu u^\lambda h_{\nu\lambda} \rangle$ | Y_{86} |
| 11 | $\langle i \chi_- h^{\mu\nu} u_\mu u_\nu + i \chi_- u^\mu u^\nu h_{\mu\nu} \rangle$ | Y_{28} | 45 | $\langle f_-^{\mu\nu} u_\mu h_\nu^\lambda u_\lambda + f_-^{\mu\nu} u^\lambda h_{\nu\lambda} u_\mu \rangle$ | Y_{89} |
| 12 | $\langle h^\lambda h^\lambda h^{\mu\nu} u_\mu u_\nu + h^\lambda h^\lambda u^\mu u^\nu h_{\mu\nu} \rangle$ | $Y_{28} - \frac{2}{N_f} Y_{30}$ | 46 | $\langle u^\mu u_\mu f_-^{\nu\lambda} f_{-\nu\lambda} \rangle$ | Y_{90} |
| 13 | $\langle i h^{\mu\nu} u_\mu \chi_- u_\nu \rangle$ | Y_{31} | 47 | $\langle f_-^{\mu\nu} u^\lambda f_{-\mu\nu} u_\lambda \rangle$ | Y_{92} |
| 14 | $\langle h^{\mu\nu} u_\mu h^\lambda h^\lambda u_\nu \rangle$ | $Y_{31} - \frac{1}{N_f} Y_{30}$ | 48 | $\langle f_-^{\mu\nu} f_{-\mu}^\lambda u_\nu u_\lambda \rangle$ | Y_{94} |
| 15 | $\langle u^\mu u_\mu \chi_- \chi_- \rangle$ | Y_{33} | 49 | $\langle f_-^{\mu\nu} f_{-\mu}^\lambda u_\lambda u_\nu \rangle$ | Y_{95} |
| 16 | $\langle i u^\mu u_\mu h^\nu h^\nu \chi_- + i u^\mu u_\mu \chi_- h^\nu h^\nu \rangle$ | $-2Y_{33} + \frac{2}{N_f} Y_{34}$ | 50 | $\langle f_-^{\mu\nu} u^\lambda f_{-\mu\lambda} u_\nu + f_-^{\mu\nu} u_\nu f_{-\mu}^\lambda u_\lambda \rangle$ | Y_{97} |
| 17 | $\langle u^\mu u_\mu h^\nu h^\lambda h^\lambda \rangle$ | $-Y_{33} + \frac{2}{N_f} Y_{34} - \frac{1}{N_f^2} Y_{36}$ | 51 | $\langle i f_+^{\mu\nu} f_{-\nu}^\lambda h_{\mu\lambda} - i f_+^{\mu\nu} h_\mu^\lambda f_{-\nu\lambda} \rangle$ | Y_{100} |
| 18 | $\langle u^\mu \chi_- u_\mu \chi_- \rangle$ | Y_{37} | 52 | $\langle i f_+^{\mu\nu} f_{-\nu}^\lambda f_{-\mu\lambda} - i f_+^{\mu\nu} f_{-\mu}^\lambda f_{-\nu\lambda} \rangle$ | Y_{101} |
| 19 | $\langle i u^\mu h^\nu h^\nu u_\mu \chi_- \rangle$ | $-Y_{37} + \frac{1}{N_f} Y_{34}$ | 53 | $\langle \chi_+ f_-^{\mu\nu} f_{-\mu\nu} \rangle$ | Y_{102} |
| 20 | $\langle u^\mu h^\nu h^\nu u_\mu h^\lambda h^\lambda \rangle$ | $-Y_{37} + \frac{2}{N_f} Y_{34} - \frac{1}{N_f^2} Y_{36}$ | 54 | $\langle f_+^{\mu\nu} f_{-\mu\nu} \chi_- - f_+^{\mu\nu} \chi_- f_{-\mu\nu} \rangle$ | Y_{104} |
| 21 | $\langle \chi_- \chi_- \chi_+ \rangle$ | Y_{39} | 55 | $\langle i f_+^{\mu\nu} f_{-\mu\nu} h^\lambda h^\lambda - i f_+^{\mu\nu} h^\lambda h^\lambda f_{-\mu\nu} \rangle$ | $-Y_{104}$ |
| 22 | $\langle i h^\mu h^\mu \chi_- \chi_+ + i h^\mu h^\mu \chi_+ \chi_- \rangle$ | $-2Y_{39} + \frac{2}{N_f} Y_{41}$ | 56 | $\langle i f_-^{\mu\nu} \chi_- u_\mu u_\nu - i f_-^{\mu\nu} u_\mu u_\nu \chi_- \rangle$ | Y_{105} |
| 23 | $\langle h^\mu h^\mu h^\nu h^\nu \chi_+ \rangle$ | $-Y_{39} + \frac{2}{N_f} Y_{41} - \frac{1}{N_f^2} Y_{42}$ | 57 | $\langle f_-^{\mu\nu} h^\lambda h^\lambda u_\mu u_\nu - f_-^{\mu\nu} u_\mu u_\nu h^\lambda h^\lambda \rangle$ | Y_{105} |
| 24 | $\langle i \chi_- \chi_+^\mu u_\mu + i \chi_- u^\mu \chi_+ u_\mu \rangle$ | Y_{43} | 58 | $\langle f_-^{\mu\nu} \chi_+ u_\mu u_\nu + f_-^{\mu\nu} u_\nu \chi_+ u_\mu \rangle$ | Y_{107} |
| 25 | $\langle h^\nu h^\nu \chi_+^\mu u_\mu + h^\nu h^\nu u^\mu \chi_+ u_\mu \rangle$ | $Y_{43} - \frac{2}{N_f} Y_{44}$ | 59 | $\langle \nabla^\mu f_-^{\nu\lambda} \nabla_\mu f_{-\nu\lambda} \rangle$ | Y_{109} |
| 26 | $\langle \chi_+^\mu \chi_+ u_\mu \rangle$ | Y_{47} | 60 | $\langle i \nabla^\lambda f_+^{\mu\nu} h_{\mu\lambda} u_\nu - i \nabla^\lambda f_+^{\mu\nu} u_\nu h_{\mu\lambda} \rangle$ | Y_{110} |
| 27 | $\langle u^\mu u_\mu u^\nu u_\nu u^\lambda u_\lambda \rangle$ | Y_{49} | 61 | $\langle i \nabla^\mu f_{+\mu}^\nu f_{-\nu}^\lambda u_\lambda - i \nabla^\mu f_{+\mu}^\nu u^\lambda f_{-\nu\lambda} \rangle$ | Y_{111} |
| 28 | $\langle u^\mu u_\mu u^\nu u^\lambda u_\lambda u_\nu \rangle$ | Y_{52} | 62 | $\langle i \nabla^\mu f_{+\mu}^\nu h_\nu^\lambda u_\lambda - i \nabla^\mu f_{+\mu}^\nu u^\lambda h_{\nu\lambda} \rangle$ | Y_{112} |
| 29 | $\langle u^\mu u_\mu u^\nu u^\lambda u_\nu u_\lambda \rangle$ | Y_{54} | 63 | $\langle i \chi_-^\mu \nabla_\mu h^\nu h^\nu \rangle$ | Z_1^a |
| 30 | $\langle u^\mu u^\nu u^\lambda u_\mu u_\nu u_\lambda \rangle$ | Y_{58} | 64 | $\langle \nabla^\mu h^\nu h^\nu \nabla_\mu h^\lambda h^\lambda \rangle$ | Z_2^b |
| 31 | $\langle u^\mu u^\nu u^\lambda u_\mu u_\lambda u_\nu \rangle$ | Y_{60} | 65 | $\langle f_+^{\mu\nu} u_\mu \chi_- u_\nu + f_+^{\mu\nu} \chi_- u_\mu u_\nu \rangle$ | Z_3^c |
| 32 | $\langle i f_+^{\mu\nu} u^\lambda u_\lambda u_\mu u_\nu + i f_+^{\mu\nu} u_\mu u_\nu u^\lambda u_\lambda \rangle$ | Y_{64} | 66 | $\langle \chi_-^\mu \chi_- u_\mu \rangle$ | (2.15) in [5] |
| 33 | $\langle i f_+^{\mu\nu} u^\lambda u_\mu u_\nu u_\lambda \rangle$ | Y_{66} | 67 | $\langle i f_+^{\mu\nu} f_{+\nu}^\lambda f_{+\mu\lambda} \rangle$ | (2.15) in [5] |
| 34 | $\langle i f_+^{\mu\nu} u_\mu u^\lambda u_\lambda u_\nu \rangle$ | Y_{67} | 68 | $\langle \nabla^\mu f_+^{\nu\lambda} \nabla_\mu f_{+\nu\lambda} \rangle$ | (2.15) in [5] |

$$^a Z_1 = -\frac{1}{2} Y_{19} - \frac{1}{2} Y_{23} + \frac{1}{N_f} Y_{24} - \frac{1}{2} Y_{33} + \frac{1}{N_f} Y_{34} - \frac{1}{2} Y_{37} - \frac{1}{2} Y_{39} + \frac{1}{N_f} Y_{41} - \frac{1}{2N_f^2} Y_{42} + \frac{1}{2} Y_{43} - \frac{1}{N_f} Y_{44} + \frac{1}{N_f} Y_{46} - Y_{47} + 4Y_{113}$$

$$^b Z_2 = -\frac{1}{2} Y_{19} - \frac{1}{2} Y_{23} + \frac{1}{N_f} Y_{24} - Y_{33} + \frac{2}{N_f} Y_{34} - Y_{37} - Y_{39} + \frac{2}{N_f} Y_{41} - \frac{1}{N_f^2} Y_{42} + Y_{43} - \frac{2}{N_f} Y_{44} + \frac{1}{N_f} Y_{46} - Y_{47} + 4Y_{113}$$

$$^c Z_3 = -Y_{66} - Y_{67} + Y_{68} + \frac{1}{2} Y_{71} - \frac{1}{2} Y_{73} + Y_{75} - 2Y_{76} + \frac{1}{2} Y_{78} - \frac{1}{2} Y_{83} - Y_{85} + \frac{1}{2} Y_{90} - \frac{1}{2} Y_{92} - Y_{94} + \frac{1}{2} Y_{97} - \frac{1}{2} Y_{100} + \frac{1}{2} Y_{101} - \frac{1}{4} Y_{104} + Y_{110} + Y_{112}$$

Our choice $F_0 = 87$ MeV is the leading order value of F_π . The relation between F_0 and F_π is given in Ref. [28] to the

$O(p^6)$ order. With our results listed in Tables I and III, the numerical results to the $O(p^6)$ order yield $F_\pi = 92.76^{+0.01}_{-0.06}$ MeV. Comparing with the previous result $F_\pi = 92.97^{+0.00}_{-0.04}$, the new result is a bit closer to that in PDG2014 [29] $F_\pi = 92.21$ MeV. From this point of view, the additional terms in (9) improve the results slightly.

TABLE III: The p^6 order LECs. They are in units of 10^{-3} GeV^{-2} . The value $\equiv 0$ means that the LECs vanish at the large N_C limit.

| i | new C_i | old C_i | j | new c_j | old c_j | i | new C_i | old C_i | j | new c_j | old c_j |
|-----|-------------------------|-------------------------|-----|-------------------------|-------------------------|-----|--------------------------|--------------------------|-----|--------------------------|--------------------------|
| 1 | $2.98^{+0.07}_{-0.13}$ | $3.79^{+0.10}_{-0.17}$ | 1 | $3.04^{+0.07}_{-0.12}$ | $3.58^{+0.09}_{-0.15}$ | 46 | $-1.67^{+0.05}_{-0.09}$ | $-0.60^{+0.02}_{-0.04}$ | 26 | $-4.82^{+0.15}_{-0.24}$ | $-1.14^{+0.05}_{-0.07}$ |
| 2 | $\equiv 0$ | $\equiv 0$ | | | | 47 | $3.10^{+0.09}_{-0.15}$ | $0.08^{+0.01}_{-0.00}$ | | | |
| 3 | $-0.05^{+0.01}_{-0.01}$ | $-0.05^{+0.01}_{-0.01}$ | 2 | $-0.09^{+0.01}_{-0.01}$ | $-0.03^{+0.01}_{-0.01}$ | 48 | $4.74^{+0.10}_{-0.17}$ | $3.41^{+0.06}_{-0.10}$ | | | |
| 4 | $2.13^{+0.06}_{-0.10}$ | $3.10^{+0.09}_{-0.15}$ | 3 | $2.15^{+0.06}_{-0.10}$ | $2.89^{+0.08}_{-0.13}$ | 49 | $\equiv 0$ | $\equiv 0$ | | | |
| 5 | $-1.28^{+0.10}_{-0.13}$ | $-1.01^{+0.08}_{-0.11}$ | 4 | $0.82^{+0.04}_{-0.03}$ | $1.21^{+0.07}_{-0.06}$ | 50 | $10.54^{+0.89}_{-1.29}$ | $8.71^{+0.78}_{-1.12}$ | 27 | $16.70^{+1.61}_{-2.30}$ | $13.57^{+1.41}_{-2.00}$ |
| 6 | $\equiv 0$ | $\equiv 0$ | | | | 51 | $-9.83^{+0.28}_{-0.24}$ | $-11.49^{+0.18}_{-0.09}$ | 28 | $5.51^{+1.22}_{-1.62}$ | $0.93^{+0.98}_{-1.25}$ |
| 7 | $\equiv 0$ | $\equiv 0$ | | | | 52 | $-6.39^{+0.77}_{-1.07}$ | $-5.04^{+0.67}_{-0.93}$ | | | |
| 8 | $2.10^{+0.15}_{-0.16}$ | $2.31^{+0.16}_{-0.18}$ | | | | 53 | $-5.36^{+0.71}_{-1.05}$ | $-11.98^{+0.87}_{-1.33}$ | 29 | $-4.77^{+0.66}_{-0.98}$ | $-11.01^{+0.81}_{-1.23}$ |
| 9 | $\equiv 0$ | $\equiv 0$ | | | | 54 | $\equiv 0$ | $\equiv 0$ | | | |
| 10 | $-0.65^{+0.05}_{-0.06}$ | $-1.05^{+0.08}_{-0.09}$ | 5 | $-0.68^{+0.05}_{-0.06}$ | $-0.98^{+0.07}_{-0.09}$ | 55 | $10.45^{+0.80}_{-1.22}$ | $16.79^{+0.96}_{-1.49}$ | 30 | $9.86^{+0.75}_{-1.14}$ | $15.72^{+0.89}_{-1.38}$ |
| 11 | $\equiv 0$ | $\equiv 0$ | | | | 56 | $4.45^{+0.03}_{-0.18}$ | $19.34^{+0.52}_{-0.98}$ | 31 | $3.27^{+0.04}_{-0.07}$ | $17.57^{+0.42}_{-0.82}$ |
| 12 | $-0.34^{+0.01}_{-0.01}$ | $-0.34^{+0.02}_{-0.01}$ | 6 | $-0.35^{+0.02}_{-0.01}$ | $-0.33^{+0.01}_{-0.01}$ | 57 | $4.72^{+1.36}_{-1.85}$ | $7.92^{+1.34}_{-1.85}$ | 32 | $4.24^{+1.32}_{-1.78}$ | $7.18^{+1.28}_{-1.76}$ |
| 13 | $\equiv 0$ | $\equiv 0$ | | | | 58 | $\equiv 0$ | $\equiv 0$ | | | |
| 14 | $-0.87^{+0.14}_{-0.21}$ | $-0.83^{+0.12}_{-0.19}$ | 7 | $-1.83^{+0.25}_{-0.35}$ | $-1.72^{+0.25}_{-0.35}$ | 59 | $-14.59^{+1.01}_{-1.55}$ | $-22.49^{+1.21}_{-1.89}$ | 33 | $-13.69^{+0.94}_{-1.44}$ | $-21.19^{+1.12}_{-1.76}$ |
| 15 | $\equiv 0$ | $\equiv 0$ | 8 | $0.91^{+0.11}_{-0.13}$ | $0.86^{+0.12}_{-0.15}$ | 60 | $\equiv 0$ | $\equiv 0$ | | | |
| 16 | $\equiv 0$ | $\equiv 0$ | | | | 61 | $2.42^{+0.19}_{-0.22}$ | $2.88^{+0.22}_{-0.26}$ | 34 | $2.40^{+0.19}_{-0.22}$ | $2.84^{+0.22}_{-0.26}$ |
| 17 | $0.17^{+0.01}_{-0.04}$ | $0.01^{+0.01}_{-0.01}$ | 9 | $-0.74^{+0.13}_{-0.18}$ | $-0.84^{+0.12}_{-0.17}$ | 62 | $\equiv 0$ | $\equiv 0$ | | | |
| 18 | $-0.60^{+0.07}_{-0.09}$ | $-0.56^{+0.09}_{-0.11}$ | | | | 63 | $2.48^{+0.21}_{-0.25}$ | $2.99^{+0.24}_{-0.30}$ | | | |
| 19 | $-0.27^{+0.09}_{-0.13}$ | $-0.48^{+0.09}_{-0.13}$ | 10 | $-0.22^{+0.07}_{-0.11}$ | $-0.37^{+0.07}_{-0.10}$ | 64 | $\equiv 0$ | $\equiv 0$ | | | |
| 20 | $0.17^{+0.02}_{-0.03}$ | $0.18^{+0.03}_{-0.04}$ | 11 | $\equiv 0$ | $\equiv 0$ | 65 | $-2.82^{+0.18}_{-0.20}$ | $-2.43^{+0.15}_{-0.16}$ | 35 | $2.16^{+0.23}_{-0.31}$ | $3.39^{+0.32}_{-0.41}$ |
| 21 | $-0.06^{+0.01}_{-0.01}$ | $-0.06^{+0.01}_{-0.01}$ | | | | 66 | $0.80^{+0.04}_{-0.07}$ | $1.71^{+0.07}_{-0.12}$ | 36 | $0.80^{+0.04}_{-0.07}$ | $1.57^{+0.06}_{-0.10}$ |
| 22 | $-0.35^{+0.20}_{-0.26}$ | $0.27^{+0.19}_{-0.25}$ | 12 | $-0.41^{+0.20}_{-0.26}$ | $0.15^{+0.18}_{-0.24}$ | 67 | $\equiv 0$ | $\equiv 0$ | | | |
| 23 | $\equiv 0$ | $\equiv 0$ | | | | 68 | $\equiv 0$ | $\equiv 0$ | | | |
| 24 | $0.87^{+0.02}_{-0.04}$ | $1.62^{+0.04}_{-0.07}$ | | | | 69 | $0.52^{+0.00}_{-0.01}$ | $-0.86^{+0.04}_{-0.06}$ | 38 | $0.60^{+0.00}_{-0.01}$ | $-0.68^{+0.03}_{-0.05}$ |
| 25 | $-3.03^{+0.41}_{-0.59}$ | $-5.98^{+0.49}_{-0.72}$ | 13 | $-3.02^{+0.39}_{-0.56}$ | $-5.39^{+0.45}_{-0.66}$ | 70 | $1.66^{+0.11}_{-0.11}$ | $1.73^{+0.08}_{-0.07}$ | 39 | $1.53^{+0.12}_{-0.12}$ | $1.81^{+0.08}_{-0.07}$ |
| 26 | $2.71^{+0.35}_{-0.54}$ | $3.35^{+0.29}_{-0.47}$ | 14 | $3.39^{+0.36}_{-0.56}$ | $4.17^{+0.30}_{-0.49}$ | 71 | $\equiv 0$ | $\equiv 0$ | | | |
| 27 | $-1.35^{+0.13}_{-0.15}$ | $-1.54^{+0.15}_{-0.18}$ | 15 | $-2.39^{+0.19}_{-0.23}$ | $-2.71^{+0.21}_{-0.25}$ | 72 | $-1.80^{+0.12}_{-0.11}$ | $-3.30^{+0.05}_{-0.00}$ | 40 | $-1.64^{+0.13}_{-0.13}$ | $-3.17^{+0.05}_{-0.02}$ |
| 28 | $0.18^{+0.00}_{-0.01}$ | $0.30^{+0.01}_{-0.01}$ | | | | 73 | $0.15^{+0.48}_{-0.62}$ | $0.50^{+0.43}_{-0.56}$ | 41 | $0.07^{+0.47}_{-0.61}$ | $0.30^{+0.42}_{-0.54}$ |
| 29 | $-0.99^{+0.21}_{-0.24}$ | $-3.08^{+0.26}_{-0.32}$ | 16 | $-0.60^{+0.19}_{-0.21}$ | $-2.22^{+0.22}_{-0.27}$ | 74 | $-3.34^{+0.11}_{-0.19}$ | $-5.07^{+0.16}_{-0.27}$ | 42 | $-3.26^{+0.10}_{-0.17}$ | $-4.74^{+0.14}_{-0.24}$ |
| 30 | $0.37^{+0.01}_{-0.02}$ | $0.60^{+0.02}_{-0.03}$ | | | | 75 | $\equiv 0$ | $\equiv 0$ | | | |
| 31 | $-0.46^{+0.07}_{-0.13}$ | $-0.63^{+0.05}_{-0.09}$ | 17 | $-0.92^{+0.15}_{-0.23}$ | $-1.10^{+0.12}_{-0.19}$ | 76 | $-1.15^{+0.26}_{-0.34}$ | $-1.44^{+0.23}_{-0.31}$ | 43 | $-1.11^{+0.25}_{-0.33}$ | $-1.29^{+0.23}_{-0.30}$ |
| 32 | $0.17^{+0.02}_{-0.03}$ | $0.18^{+0.03}_{-0.04}$ | 18 | $0.42^{+0.06}_{-0.08}$ | $0.43^{+0.07}_{-0.08}$ | 77 | $\equiv 0$ | $\equiv 0$ | | | |
| 33 | $-0.05^{+0.02}_{-0.05}$ | $0.09^{+0.00}_{-0.03}$ | 19 | $0.29^{+0.07}_{-0.13}$ | $0.41^{+0.06}_{-0.10}$ | 78 | $8.82^{+0.80}_{-1.21}$ | $17.51^{+1.02}_{-1.59}$ | 44 | $8.19^{+0.74}_{-1.12}$ | $16.16^{+0.94}_{-1.45}$ |
| 34 | $0.66^{+0.18}_{-0.29}$ | $1.59^{+0.17}_{-0.17}$ | 20 | $0.74^{+0.18}_{-0.28}$ | $1.56^{+0.10}_{-0.17}$ | 79 | $5.86^{+0.14}_{-0.12}$ | $-0.56^{+0.30}_{-0.40}$ | 45 | $6.09^{+0.13}_{-0.11}$ | $0.26^{+0.26}_{-0.34}$ |
| 35 | $0.10^{+0.09}_{-0.12}$ | $0.17^{+0.12}_{-0.17}$ | 21 | $0.22^{+0.14}_{-0.19}$ | $0.29^{+0.18}_{-0.24}$ | 80 | $1.01^{+0.05}_{-0.04}$ | $0.87^{+0.04}_{-0.03}$ | 46 | $1.09^{+0.05}_{-0.05}$ | $0.85^{+0.04}_{-0.02}$ |
| 36 | $\equiv 0$ | $\equiv 0$ | | | | 81 | $\equiv 0$ | $\equiv 0$ | | | |

| i | new C_i | old C_i | j | new c_j | old c_j | i | new C_i | old C_i | j | new c_j | old c_j |
|-----|-------------------------|-------------------------|-----|-------------------------|-------------------------|-----|-------------------------|-------------------------|-----|-------------------------|-------------------------|
| 37 | $-0.60^{+0.07}_{-0.09}$ | $-0.56^{+0.09}_{-0.11}$ | | | | 82 | $-4.58^{+0.24}_{-0.38}$ | $-7.13^{+0.32}_{-0.51}$ | 47 | $-4.26^{+0.22}_{-0.35}$ | $-6.73^{+0.29}_{-0.47}$ |
| 38 | $0.47^{+0.04}_{-0.02}$ | $0.41^{+0.08}_{-0.07}$ | 22 | $-1.27^{+0.19}_{-0.26}$ | $-1.32^{+0.18}_{-0.25}$ | 83 | $-1.74^{+0.17}_{-0.22}$ | $0.07^{+0.20}_{-0.27}$ | 48 | $-1.71^{+0.17}_{-0.21}$ | $-0.22^{+0.18}_{-0.25}$ |
| 39 | $\equiv 0$ | $\equiv 0$ | 23 | $0.91^{+0.11}_{-0.13}$ | $0.86^{+0.12}_{-0.15}$ | 84 | $\equiv 0$ | $\equiv 0$ | | | |
| 40 | $-4.98^{+0.14}_{-0.25}$ | $-6.35^{+0.18}_{-0.32}$ | 24 | $0.00^{+0.01}_{-0.03}$ | $-4.84^{+0.14}_{-0.25}$ | 85 | $-0.96^{+0.04}_{-0.03}$ | $-0.82^{+0.03}_{-0.02}$ | 49 | $-1.07^{+0.05}_{-0.04}$ | $-0.78^{+0.03}_{-0.01}$ |
| 41 | $\equiv 0$ | $\equiv 0$ | | | | 86 | $\equiv 0$ | $\equiv 0$ | | | |
| 42 | $1.88^{+0.03}_{-0.06}$ | $0.60^{+0.00}_{-0.00}$ | | | | 87 | $4.79^{+0.29}_{-0.46}$ | $7.57^{+0.37}_{-0.60}$ | 50 | $4.35^{+0.26}_{-0.42}$ | $7.18^{+0.34}_{-0.55}$ |
| 43 | $\equiv 0$ | $\equiv 0$ | | | | 88 | $-1.69^{+0.68}_{-0.93}$ | $-5.47^{+0.73}_{-1.03}$ | 51 | $-1.36^{+0.65}_{-0.89}$ | $-4.85^{+0.69}_{-0.97}$ |
| 44 | $1.73^{+0.07}_{-0.14}$ | $6.32^{+0.20}_{-0.36}$ | 25 | $4.88^{+0.16}_{-0.27}$ | $6.03^{+0.19}_{-0.33}$ | 89 | $17.27^{+1.11}_{-1.77}$ | $34.74^{+1.61}_{-2.62}$ | 52 | $15.84^{+1.00}_{-1.60}$ | $32.19^{+1.46}_{-2.37}$ |
| 45 | $\equiv 0$ | $\equiv 0$ | | | | 90 | $2.32^{+0.44}_{-0.55}$ | $2.44^{+0.38}_{-0.46}$ | 53 | $2.28^{+0.44}_{-0.55}$ | $2.51^{+0.37}_{-0.46}$ |

C. Anomaly

Following the same procedures as for the normal parts, the anomalous LECs can also be revised. The $O(p^4)$ CL are Wess-Zumino terms, that had been obtained from the first term in (1) [20, 30]. The additional terms in (9) must vanish to the $O(p^4)$ order., thereby imposing another requirement. We checked our calculations and verified this requirement. From another point of view, we checked that the terms with $n = 2$ in (1) and $n = 1$ in (4) were suitable.

To the $O(p^6)$ order, without the equations of motion, in the large N_C limit, the n -flavor CL is

$$\mathcal{L}_6^W = \sum_{n=1}^{23} \tilde{K}_n^W \tilde{O}_n^W. \quad (24)$$

These \tilde{O}_n^W and their relations with O_n^W in Ref. [8] are listed in Table VI in Ref. [20]. The analytical results for $\Delta \tilde{K}_n^W$ are listed in (B1) in Appendix B. C_3^W , C_{18}^W and c_{12}^W vanish in the large N_C limit. The nonzero numerical results are listed in the second column in Table IV for three flavors and in the second row in Table V for two flavours.

$$C_{\Lambda=1\text{GeV}}^W \begin{vmatrix} C_{\Lambda=1.1\text{GeV}}^W - C_{\Lambda=1\text{GeV}}^W \\ C_{\Lambda=0.9\text{GeV}}^W - C_{\Lambda=1\text{GeV}}^W \end{vmatrix}, \quad c_{\Lambda=1\text{GeV}}^W \begin{vmatrix} c_{\Lambda=1.1\text{GeV}}^W - c_{\Lambda=1\text{GeV}}^W \\ c_{\Lambda=0.9\text{GeV}}^W - c_{\Lambda=1\text{GeV}}^W \end{vmatrix}.$$

V. COMPARISONS

In this section, we shall gather the LECs to the $O(p^6)$ order given in the literature to provide a means to assess our new results. It is just only an update of [18] but also includes some new results. Usually, these LECs are given as dimensionless parameters with the convention of $C_i^r \equiv C_i F_0^2$ or $c_i^r \equiv c_i F_0^2$. The following values of the physical constants come from the central values of PDG2014 [29],

$$m_{\pi^\pm} = 139.57018\text{MeV}, m_{\pi^0} = 134.9766\text{MeV}, F_\pi = 92.21\text{MeV}, m_{K^\pm} = 493.677\text{MeV}. \quad (25)$$

Some of these values are needed in certain parts of the calculations.

A. $\pi\pi$ and πK scattering

In $\pi\pi$ scattering there exist six additional constants, r_i^r , $i = 1, 2, 3, 4, 5, 6$. Their relationship to LECs can be found in Eq. (5.3) in [38]. Ref. [38] also gives their values obtained using two theoretical methods, resonance-saturation (RS)[39] and pure dimensional analysis (ND) which only accounts for the order of magnitude. Ref. [40, 41] also give the r_i^r parameters, all of which are listed in Table. VI. Our new $r_{4,5,6}^r$ only change slightly, the error bars from the references being also small. They are nonetheless in good agreement. However, in the references, $r_{1,2,3}^r$ have large error bars, and hence our new results can potentially change signs.

TABLE IV: The nonzero values of the p^6 order anomalous LECs C_i^W in three flavors. They are in units of 10^{-3} GeV^{-2} . The forth column to the eighth column contain the results given in [31]: (I)-ChPT, (II)-VMD, (III)-ChPT (extrapolation), (IV)-CQM, (V)-CQM (extrapolation).

| n | new | old[20] | (I)[31] | (II)[31] | (III)[31] | (IV)[31] | (V)[31] | [17] | [32-37] |
|-----|-------------------------|-------------------------|-------------------------------------|----------|------------------|------------------------------------|-------------------|--------|---|
| 1 | $2.90^{+0.49}_{-0.69}$ | $4.97^{+0.55}_{-0.79}$ | | | | | | | |
| 2 | $-1.79^{+0.09}_{-0.11}$ | $-1.43^{+0.10}_{-0.12}$ | -0.32 ± 10.4 | | 0.78 ± 12.7 | 4.96 ± 9.70 | -0.074 ± 13.3 | | |
| 4 | $-1.89^{+0.19}_{-0.24}$ | $-0.96^{+0.22}_{-0.29}$ | 0.28 ± 9.19 | | 0.67 ± 10.9 | 6.32 ± 6.09 | -0.55 ± 9.05 | | |
| 5 | $1.56^{+0.29}_{-0.41}$ | $3.26^{+0.34}_{-0.49}$ | 28.5 ± 28.83 | | 9.38 ± 152.2 | 33.05 ± 28.66 | 34.51 ± 41.13 | | |
| 6 | $0.72^{+0.02}_{-0.04}$ | $0.91^{+0.03}_{-0.04}$ | | | | | | | |
| 7 | $2.02^{+0.23}_{-0.29}$ | $1.68^{+0.24}_{-0.31}$ | 0.013 ± 1.17 20.3 ± 18.7 | | | 0.51 ± 0.06 | | | 0.1 ± 1.2 1.0^a 0.35 ± 0.07 0.58 ± 0.20 5.0^a |
| 8 | $0.52^{+0.01}_{-0.03}$ | $0.41^{+0.01}_{-0.02}$ | 0.76 ± 0.18 | | | | | | |
| 9 | $1.21^{+0.02}_{-0.03}$ | $1.15^{+0.03}_{-0.03}$ | | | | | | | |
| 10 | $-0.14^{+0.01}_{-0.01}$ | $-0.18^{+0.01}_{-0.01}$ | | | | | | | |
| 11 | $-1.39^{+0.07}_{-0.09}$ | $-1.15^{+0.08}_{-0.10}$ | -6.37 ± 4.54 | | | -0.00143 ± 0.03 | | | 0.68 ± 0.21 |
| 12 | $-4.05^{+0.12}_{-0.20}$ | $-5.13^{+0.15}_{-0.25}$ | | | | | | -2.1 | |
| 13 | $-6.81^{+0.33}_{-0.07}$ | $-6.37^{+0.31}_{-0.06}$ | -74.09 ± 55.89 | -20.00 | -8.44 ± 69.9 | 14.15 ± 15.22 | -7.46 ± 19.62 | -8.8 | |
| 14 | $-2.48^{+0.12}_{-0.07}$ | $-2.00^{+0.10}_{-0.12}$ | 29.99 ± 11.14 | -6.01 | 0.72 ± 15.3 | 10.23 ± 7.56 | -0.58 ± 9.77 | -1.3 | |
| 15 | $2.35^{+0.11}_{-0.05}$ | $4.17^{+0.12}_{-0.20}$ | -25.3 ± 23.93 | 2.00 | -3.10 ± 28.6 | 19.70 ± 7.49 | 8.89 ± 9.72 | 4.4 | |
| 16 | $1.79^{+0.09}_{-0.09}$ | $3.58^{+0.17}_{-0.17}$ | | | | | | -0.2 | |
| 17 | $0.71^{+0.02}_{-0.03}$ | $1.98^{+0.06}_{-0.10}$ | | | | | | -0.1 | |
| 19 | $0.56^{+0.02}_{-0.03}$ | $0.29^{+0.01}_{-0.01}$ | | | | | | -7.0 | |
| 20 | $-1.02^{+0.03}_{-0.05}$ | $1.82^{+0.05}_{-0.09}$ | | | | | | -0.4 | |
| 21 | $3.11^{+0.09}_{-0.15}$ | $2.48^{+0.07}_{-0.12}$ | | | | | | 2.6 | |
| 22 | $4.72^{+0.14}_{-0.23}$ | $5.01^{+0.14}_{-0.24}$ | 6.52 ± 0.78 5.07 ± 0.71 | 8.01 | | 3.94 ± 0.43 3.94 ± 0.43 | | 7.9 | 5.4 ± 0.8 $6.71, 6.21, 4.45^b$ |
| 23 | $2.80^{+0.08}_{-0.14}$ | $2.74^{+0.08}_{-0.13}$ | | | | | | 0.9 | |

^a This result is just the absolute value given in [35].

^b These result are in [37] by different inputs.

TABLE V: The nonzero values of the p^6 order anomalous LECs c_i^W in two flavors. They are in units of 10^{-3} GeV^{-2} .

| | c_1^W | c_2^W | c_3^W | c_4^W | c_5^W | c_6^W | c_7^W | c_8^W | c_9^W | c_{10}^W | c_{11}^W | c_{13}^W |
|---------|-------------------------|-------------------------|------------------------|------------------------|-------------------------|------------------------|-------------------------|------------------------|------------------------|-------------------------|------------------------|-------------------------|
| new | $-1.81^{+0.09}_{-0.11}$ | $-1.61^{+0.08}_{-0.09}$ | $3.61^{+0.19}_{-0.22}$ | $0.80^{+0.04}_{-0.04}$ | $-1.40^{+0.07}_{-0.09}$ | $0.52^{+0.26}_{-0.35}$ | $-0.41^{+0.01}_{-0.02}$ | $0.21^{+0.01}_{-0.01}$ | $6.49^{+0.18}_{-0.30}$ | $-6.15^{+0.17}_{-0.29}$ | $4.63^{+0.13}_{-0.22}$ | $-9.25^{+0.26}_{-0.43}$ |
| old[20] | $-1.46^{+0.10}_{-0.12}$ | $-1.25^{+0.09}_{-0.11}$ | $2.96^{+0.20}_{-0.25}$ | $0.63^{+0.04}_{-0.05}$ | $-1.17^{+0.08}_{-0.10}$ | $0.77^{+0.26}_{-0.36}$ | $-0.04^{+0.00}_{-0.00}$ | $0.02^{+0.00}_{-0.00}$ | $8.19^{+0.23}_{-0.38}$ | $-8.73^{+0.24}_{-0.41}$ | $4.85^{+0.13}_{-0.23}$ | $-9.70^{+0.27}_{-0.45}$ |

Furthermore, Ref. [42] introduces some coefficients, such as c_{20}^+ , c_{01}^+ , c_{10}^+ , in πK Scattering. Table VII lists all these coefficients and some LECs which were obtained in Ref. [43] from different models. Whereas the new results on the left-hand side of Table VII are smaller and approach their results, results on the right-hand side of of Table VII seem worse. This may be because of propagation errors from the complex relations between c^+ and C_i .

B. Form factors

Ref. [38] also estimates the expressions of the vector form factor, the scalar form factor, two form factors of $\pi(p) \rightarrow e\nu\gamma(q)$ with LECs, and [44, 45] give some LECs using measurements of the pion scalar form factor, K_{l3} and the πK form factors. Ref. [40] gives one form factor from $\pi\pi$ -scattering. All of them are listed in Table VIII. Ref. [46] extrapolates the lattice data on the scalar $K\pi$ form factor to obtain some LECs; the results are listed in Table IX. Two of these results, r_{S2}^r and $2C_{12}^r + C_{34}^r$, seem better, as their signs have been changed.

Furthermore in Fig.1, we compare the experimental data [47-53] for the vector form factors collected in Figure 4 and 5 in [28] with our old and new results. In obtaining our numerical predictions, we have exploited the formula

TABLE VI: The obtained values for the combinations of the p^6 order LECs from $\pi\pi$ scattering and our work. The coefficients in the table are in units of 10^{-4} .

| | r_1^r | r_2^r | r_3^r | r_4^r | r_5^r | r_6^r |
|-----------------|-------------------------|-------------------------|-------------------------|---------------------------|------------------------|--------------------------|
| RS in Ref.[38] | -0.6 | 1.3 | -1.7 | -1.0 | 1.1 | 0.3 |
| ND in Ref.[38] | 80 | 40 | 20 | 3 | 6 | 2 |
| set C (n=5)[40] | $-14 \pm 17 \pm 3$ | $22 \pm 16 \pm 4$ | $-3 \pm 1 \pm 3$ | $-0.22 \pm 0.13 \pm 0.05$ | $0.9 \pm 0.1 \pm 0.5$ | $0.25 \pm 0.01 \pm 0.05$ |
| set C (n=3)[40] | $-20 \pm 17 \pm 3$ | $7 \pm 10 \pm 4$ | $-4 \pm 1 \pm 3$ | $0.13 \pm 0.13 \pm 0.05$ | $0.9 \pm 0.1 \pm 0.5$ | $0.25 \pm 0.01 \pm 0.05$ |
| Ref. [41] | | 18 | 0.9 | -1.9 | | |
| Old[18] | $-9.32_{-2.62}^{+3.51}$ | $8.93_{-4.27}^{+3.12}$ | $-3.06_{+1.11}^{-0.81}$ | $-0.12_{-0.29}^{+0.22}$ | $0.87_{-0.06}^{+0.04}$ | $0.42_{-0.03}^{+0.02}$ |
| new | $0.11_{+3.27}^{-2.50}$ | $-2.84_{-3.94}^{+2.95}$ | $1.03_{+0.92}^{-0.69}$ | $-0.63_{-0.28}^{+0.21}$ | $0.37_{-0.04}^{+0.02}$ | $0.28_{-0.02}^{+0.01}$ |

TABLE VII: The obtained values for the combinations of the p^6 order LECs from πK scattering and our work. The LECs in the l.h.s. of the table are in units of 10^{-4}GeV^{-2} .

| | $C_1 + 4C_3$ | C_2 | $C_4 + 3C_3$ | $C_1 + 4C_3 + 2C_2$ | | $c_{20}^+ \frac{m_\pi^4}{F_\pi^4}$ | $c_{01}^+ \frac{m_\pi^2}{F_\pi^4}$ | $c_{10}^- \frac{m_\pi^3}{F_\pi^4}$ |
|--------------------------------------|----------------------|---------------------|----------------------|----------------------|-----------------|------------------------------------|------------------------------------|------------------------------------|
| input $c_{30}^+, c_{11}^+, c_{20}^-$ | 20.7 ± 4.9 | -9.2 ± 4.9 | 9.9 ± 2.5 | 2.3 ± 10.8 | | | | |
| input $c_{30}^+, c_{11}^+, c_{01}^-$ | 28.1 ± 4.9 | -7.4 ± 4.9 | 21.0 ± 2.5 | 13.4 ± 10.8 | Dispersive | 0.024 ± 0.006 | 2.07 ± 0.10 | 0.31 ± 0.01 |
| $\pi\pi$ amplitude | | | 23.5 ± 2.3 | 18.8 ± 7.2 | | | | |
| Resonance model | 7.2 | -0.5 | 10.0 | 6.2 | Resonance model | 0.003 | 3.8 | 0.09 |
| Old [18] | $35.9_{-2.1}^{+1.3}$ | $0.0_{-0.0}^{+0.0}$ | $29.5_{-1.9}^{+1.1}$ | $35.9_{-2.1}^{+1.3}$ | Old [18] | $0.006_{+0.003}^{-0.002}$ | $-0.159_{-0.178}^{+0.133}$ | $0.020_{-0.050}^{+0.037}$ |
| New | $27.8_{-1.8}^{+1.1}$ | $0.0_{-0.0}^{+0.0}$ | $19.8_{-1.4}^{+0.9}$ | $27.8_{-1.8}^{+1.1}$ | New | $0.016_{+0.002}^{-0.002}$ | $-0.474_{-0.186}^{+0.142}$ | $-0.082_{-0.047}^{+0.036}$ |

given by Eq.(3.16) in [28] which especially depends on $O(p^6)$ LECs through r_{V1}^r and r_{V2}^r defined in [38], and we input the formula with the old and new $O(p^4)$ and $O(p^6)$ LECs.

From Fig.1, we see that at both $O(p^4)$ and $O(p^6)$, for the space-like form factors, the new line is higher than the old ones, whereas for the time-like form factors, the new line are lower than old ones. The new results are slightly worse than the old ones. Nevertheless, considering the experimental errors, they are all consistent with the experimental data.

TABLE VIII: The obtained values for the combinations of the p^6 order LECs appear in vector and scalar form factor of pion. the coefficients in the table are in units of 10^{-4} .

| | New | Old[18] | Ref.[38] | | New | Old[18] | Ref.[38] | Ref.[40] | | New | Old[18] | Ref.[38] |
|------------|-------------------------|-------------------------|----------|------------|-------------------------|------------------------|----------|-----------------|------------|-------------------------|-------------------------|----------|
| r_{V1}^r | $-1.60_{-0.41}^{+0.32}$ | $-2.13_{-0.39}^{+0.30}$ | -2.5 | r_{S2}^r | $-0.86_{+0.00}^{+0.00}$ | $0.07_{-0.08}^{+0.05}$ | -0.3 | $1 \pm 4 \pm 1$ | r_{A1}^r | $1.29_{-0.06}^{+0.05}$ | $1.14_{-0.09}^{+0.07}$ | -0.5 |
| r_{V2}^r | $1.10_{-0.10}^{+0.07}$ | $2.23_{-0.16}^{+0.10}$ | 2.6 | r_{S3}^r | $0.21_{+0.01}^{-0.01}$ | $0.20_{+0.01}^{-0.01}$ | 0.6 | | r_{A2}^r | $-0.72_{+0.10}^{-0.07}$ | $-0.38_{+0.08}^{-0.06}$ | 1.1 |

| | New | Old[18] | Ref.[44] | | New | Old[18] | Ref.[45] |
|------------------------|----------------------------|----------------------------|------------------|-----------------------|----------------------------|----------------------------|--------------------------------|
| C_{12}^r | $-0.026_{-0.001}^{+0.001}$ | $-0.026_{-0.001}^{+0.001}$ | -0.1 | C_{12}^r | $-0.026_{-0.001}^{+0.001}$ | $-0.026_{-0.001}^{+0.001}$ | $(0.3 \pm 5.4) \times 10^{-3}$ |
| $2C_{12}^r + C_{34}^r$ | $-0.001_{+0.020}^{-0.012}$ | $0.068_{+0.010}^{-0.006}$ | -0.10 ± 0.17 | $C_{12}^r + C_{34}^r$ | $0.025_{+0.021}^{-0.013}$ | $0.094_{+0.011}^{-0.007}$ | $(3.2 \pm 1.5) \times 10^{-2}$ |

TABLE IX: The LECs come from extrapolating the lattice data on the scalar $K\pi$ form factor. They are in units of 10^{-4}GeV^{-2} .

| | C_{12} | C_{34} | C_{14} | $2C_{17}$ |
|----------|-------------------------|------------------------|-------------------------|------------------------|
| Ref.[46] | 5.74 ± 0.95 | 1.07 ± 0.96 | 0.71 ± 1.42 | 1.92 ± 3.36 |
| Old[18] | $-0.34^{+0.02}_{-0.01}$ | $1.59^{+0.10}_{-0.17}$ | $-0.83^{+0.12}_{-0.19}$ | $0.03^{+0.02}_{-0.02}$ |
| New | $-0.34^{+0.01}_{-0.01}$ | $0.66^{+0.18}_{-0.29}$ | $-0.87^{+0.14}_{-0.21}$ | $0.35^{+0.03}_{-0.08}$ |

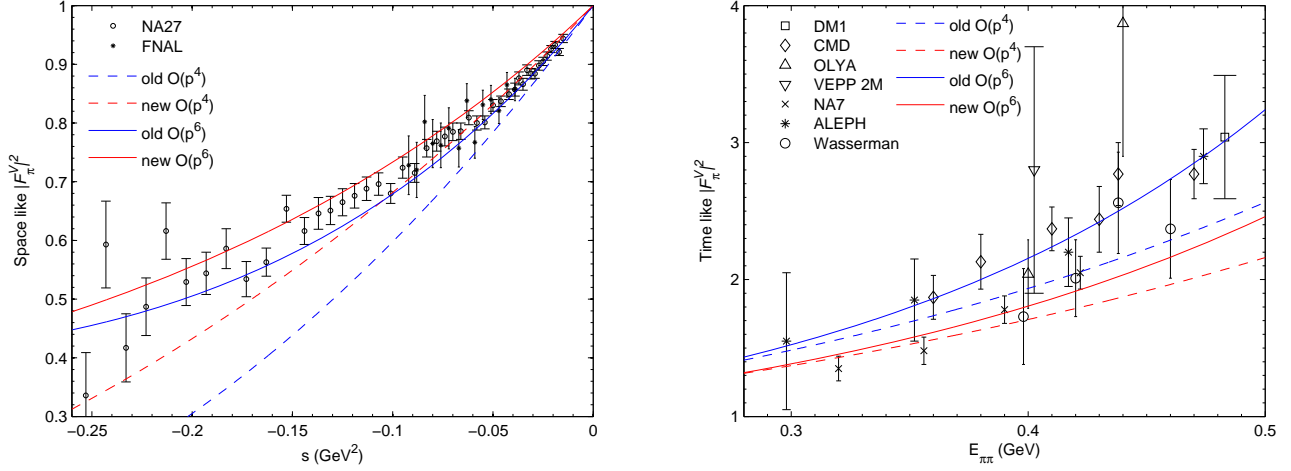


FIG. 1: The space like and time like data for the vector form factor with the old and new results.

C. Photon-photon collisions

Ref. [54, 55] introduce some parameters by $\gamma\gamma \rightarrow \pi^0\pi^0$ and $\gamma\gamma \rightarrow \pi^+\pi^-$. These parameters are also all related to LECs. They are listed in Table X. These results are nearly unchanged, with a_2^r and b^r still having opposite signs.

TABLE X: The obtained values for the combinations of the p^6 order LECs appear in photon-photon collisions.

| | New | Old[18] | Ref.[54] | | New | Old[18] | Ref.[55] |
|---------|-------------------------|-------------------------|-------------|---------|-------------------------|-------------------------|----------|
| a_1^r | $-8.15^{+0.85}_{-1.17}$ | $-5.65^{+0.91}_{-1.23}$ | -14 ± 5 | a_1^r | $-5.46^{+0.12}_{-0.07}$ | $-5.86^{+0.49}_{-0.58}$ | -3.2 |
| a_2^r | $3.94^{+0.03}_{-0.06}$ | $3.79^{+0.02}_{-0.05}$ | 7 ± 3 | a_2^r | $-1.73^{+0.09}_{-0.16}$ | $-0.98^{+0.07}_{-0.12}$ | 0.7 |
| b^r | $1.87^{+0.06}_{-0.10}$ | $1.66^{+0.05}_{-0.09}$ | 3 ± 1 | b^r | $-0.20^{+0.02}_{-0.02}$ | $-0.23^{+0.01}_{-0.02}$ | 0.4 |

D. Radiative pion decay

Ref. [32] gives a group of LECs to the $O(p^6)$ order. They are listed in Table XI. Most of the new results are fitted better than the old ones, with the exception of C_{88}^r .

TABLE XI: The obtained values for the combinations of the p^6 order LECs from pion radiative decay and our work. The coefficients in the table are in units of 10^{-5} .

| | C_{12}^r | C_{13}^r | C_{61}^r | C_{62}^r | $2C_{63}^r - C_{65}^r$ | C_{64}^r |
|----------|-------------------------|------------------------|------------------------|-------------------------|------------------------|-------------------------|
| Ref.[32] | -0.6 ± 0.3 | 0 ± 0.2 | 1.0 ± 0.3 | 0 ± 0.2 | 1.8 ± 0.7 | 0 ± 0.2 |
| Old[18] | $-0.26^{+0.01}_{-0.01}$ | $0.0^{+0.0}_{-0.0}$ | $2.18^{+0.17}_{-0.20}$ | $0.0^{+0.0}_{-0.0}$ | $6.36^{+0.42}_{-0.56}$ | $0.0^{+0.0}_{-0.0}$ |
| New | $-0.26^{+0.01}_{-0.01}$ | $0.00^{+0.00}_{-0.00}$ | $1.83^{+0.14}_{-0.17}$ | $0.00^{+0.00}_{-0.00}$ | $5.89^{+0.45}_{-0.53}$ | $0.00^{+0.00}_{-0.00}$ |
| | C_{78}^r | C_{80}^r | C_{81}^r | C_{82}^r | C_{87}^r | C_{88}^r |
| Ref.[32] | 10.0 ± 3.0 | 1.8 ± 0.4 | 0 ± 0.2 | -3.5 ± 1.0 | 3.6 ± 1.0 | -3.5 ± 1.0 |
| Old[18] | $13.26^{+0.77}_{-1.20}$ | $0.66^{+0.03}_{-0.02}$ | $0.0^{+0.0}_{-0.0}$ | $-5.39^{+0.24}_{-0.39}$ | $5.73^{+0.28}_{-0.45}$ | $-4.14^{+0.55}_{-0.78}$ |
| New | $6.68^{+0.60}_{-0.91}$ | $0.77^{+0.04}_{-0.03}$ | $0.00^{+0.00}_{-0.00}$ | $-3.47^{+0.18}_{-0.29}$ | $3.63^{+0.22}_{-0.35}$ | $-1.28^{+0.52}_{-0.70}$ |

E. holography

Ref. [17] gives almost all LECs without scalar and pseudoscalar fields in the large N_C limit from a class of holographic theories, the results are listed in Table XII. The new results still produce some large differences in these LECs, but some LECs retain the same sign, such as C_{69} and $C_{79} - \frac{1}{2}C_{90}$. Some differences may come from the error associated with C_{90} , the source of which needs to be further checked.

TABLE XII: The p^6 order LECs from Cosh holographic models [17] and our work. They are in units of 10^{-3}GeV^{-2} .

| | C_1 | C_3 | C_4 | C_{40} | C_{42} | C_{44} | C_{46} | C_{47} |
|----------|------------------------------|------------------------------|-------------------------|------------------------------|------------------------------|------------------------------|-------------------------|------------------------------|
| Ref.[17] | -0.3 | 0.3 | 0 | 0.2 | 2.2 | -5.5 | -3.2 | 6.2 |
| Old[18] | $3.79^{+0.10}_{-0.17}$ | $-0.05^{+0.01}_{-0.01}$ | $3.10^{+0.09}_{-0.15}$ | $-6.35^{+0.18}_{-0.32}$ | $0.60^{+0.00}_{-0.00}$ | $6.32^{+0.20}_{-0.36}$ | $-0.60^{+0.02}_{-0.04}$ | $0.08^{+0.01}_{-0.00}$ |
| New | $2.98^{+0.07}_{-0.13}$ | $-0.05^{+0.01}_{-0.01}$ | $2.13^{+0.06}_{-0.10}$ | $-4.98^{+0.14}_{-0.25}$ | $1.88^{+0.03}_{-0.06}$ | $1.73^{+0.07}_{-0.14}$ | $-1.67^{+0.05}_{-0.09}$ | $3.10^{+0.09}_{-0.15}$ |
| | C_{48} | $C_{50} + C_{90}$ | $C_{51} + C_{90}$ | $C_{52} - C_{90}$ | $C_{53} - \frac{1}{2}C_{90}$ | $C_{55} + \frac{1}{2}C_{90}$ | $C_{56} - C_{90}$ | $C_{57} + 2C_{90}$ |
| Ref.[17] | 5.8 | 19.1 | 5.2 | -11.6 | -8.8 | 16.7 | 7.1 | 17.2 |
| Old[18] | $3.41^{+0.06}_{-0.10}$ | $11.16^{+0.40}_{-0.66}$ | $-9.04^{+0.20}_{-0.38}$ | $-7.48^{+0.29}_{-0.47}$ | $-13.21^{+0.68}_{-1.10}$ | $18.01^{+0.77}_{-1.26}$ | $16.89^{+0.89}_{-1.44}$ | $12.80^{+0.58}_{-0.92}$ |
| New | $4.74^{+0.10}_{-0.17}$ | $12.86^{+0.45}_{-0.75}$ | $-7.50^{+0.15}_{-0.30}$ | $-8.71^{+0.33}_{-0.53}$ | $-6.52^{+0.49}_{-0.78}$ | $11.61^{+0.58}_{-0.95}$ | $2.13^{+0.47}_{-0.73}$ | $9.36^{+0.48}_{-0.76}$ |
| | $C_{59} - \frac{1}{2}C_{90}$ | C_{66} | C_{69} | $C_{70} - \frac{1}{2}C_{90}$ | $C_{72} + \frac{1}{2}C_{90}$ | $C_{73} + C_{90}$ | C_{74} | $C_{76} - \frac{1}{2}C_{90}$ |
| Ref.[17] | -20.1 | -0.3 | 0.3 | 5.3 | -4.7 | -4.4 | -19.0 | 11.1 |
| Old[18] | $-23.71^{+1.02}_{-1.66}$ | $1.71^{+0.07}_{-0.12}$ | $-0.86^{+0.04}_{-0.06}$ | $0.51^{+0.11}_{-0.16}$ | $-2.08^{+0.14}_{-0.23}$ | $2.94^{+0.05}_{-0.10}$ | $-5.07^{+0.16}_{-0.27}$ | $-2.66^{+0.05}_{-0.08}$ |
| New | $-15.75^{+0.79}_{-1.28}$ | $0.80^{+0.04}_{-0.07}$ | $0.52^{+0.00}_{-0.01}$ | $0.49^{+0.11}_{-0.16}$ | $-0.64^{+0.10}_{-0.16}$ | $2.48^{+0.04}_{-0.07}$ | $-3.34^{+0.11}_{-0.19}$ | $-2.31^{+0.04}_{-0.07}$ |
| | $C_{78} + \frac{1}{2}C_{90}$ | $C_{79} - \frac{1}{2}C_{90}$ | C_{87} | $C_{88} - C_{90}$ | C_{89} | | | |
| Ref.[17] | 16.1 | 4.1 | 6.8 | -5.2 | 29.2 | | | |
| Old[18] | $18.74^{+0.83}_{-1.36}$ | $-1.78^{+0.11}_{-0.17}$ | $7.57^{+0.37}_{-0.60}$ | $-7.91^{+0.35}_{-0.57}$ | $34.74^{+1.61}_{-2.62}$ | | | |
| New | $9.99^{+0.58}_{-0.93}$ | $4.70^{+0.08}_{-0.15}$ | $4.79^{+0.29}_{-0.46}$ | $-4.01^{+0.24}_{-0.38}$ | $17.27^{+1.11}_{-1.77}$ | | | |

F. Other results

Aside from the above results, there are more LECs given using these different method. Most of them are from resonance approximations, but our results do not rely on the assumption of the presence of resonances. In this subsection, we list the values we gathered from the literature and compare with ours.

Table XIII lists some LECs from resonance estimates in [12]; Table XIV lists other resonance-estimated LECs in [13]; Table XV lists the LECs from sum rules in [14]; C_{87}^r values were obtained from many references, and are listed Table XVI; the other LECs obtained from different models in different references are listed in Table XVII. Except for $C_{14}^r + C_{15}^r$ in Table XVII, all the LECs retain their signs and the same orders of magnitudes, and almost all new results are in closer correspondence with others.

In Table XVIII, we checked the relations for the large N_C limit given in Ref. [56].

$$C_{20} = -3C_{21} = C_{32} = \frac{1}{6}C_{35}, \quad C_{24} = 6C_{28} = 3C_{30}. \quad (26)$$

Because we only calculate a part of the large- N_C expression in (1), not all of the LECs satisfy the relations.

For the anomalous parts, we collect the results in Table IV. The anomalous results are less than the normal ones, and the differences between each are slightly larger than the normal ones.

TABLE XIII: The obtained values for the p^6 order LECs in Ref.[12] and our works They are in units of 10^{-3} GeV^{-2} .

| | C_{14} | C_{19} | C_{38} | C_{61} | C_{80} | C_{87} |
|----------|-------------------------|-------------------------|------------------------|------------------------|------------------------|------------------------|
| Ref.[12] | -4.3 | -2.8 | 1.2 | 1.9 | 1.9 | 7.6 |
| Old[18] | $-0.83^{+0.12}_{-0.19}$ | $-0.48^{+0.09}_{-0.13}$ | $0.41^{+0.08}_{-0.07}$ | $2.88^{+0.22}_{-0.26}$ | $0.87^{+0.04}_{-0.03}$ | $7.57^{+0.37}_{-0.60}$ |
| New | $-0.87^{+0.14}_{-0.21}$ | $-0.27^{+0.09}_{-0.13}$ | $0.47^{+0.04}_{-0.02}$ | $2.42^{+0.19}_{-0.22}$ | $1.01^{+0.05}_{-0.04}$ | $4.79^{+0.29}_{-0.46}$ |

TABLE XIV: The obtained values for the p^6 order LECs from resonance Lagrangian given by Ref.[13] and our work The coefficients in the table are in units of $10^{-4}/F_0^2$.

| | C_{78} | C_{82} | C_{87} | C_{88} | C_{89} | C_{90} |
|-------------------------|---------------------------|----------------------------|---------------------------|----------------------------|---------------------------|---------------------------|
| Lowest Meson Dominance | 1.09 | -0.36 | 0.40 | -0.52 | 1.97 | 0.0 |
| Resonance Lagrangian I | 1.09 | -0.29 | 0.47 | -0.16 | 2.29 | 0.33 |
| Resonance Lagrangian II | 1.49 | -0.39 | 0.65 | -0.14 | 3.22 | 0.51 |
| Old[18] | $1.326^{+0.077}_{-0.120}$ | $-0.539^{+0.024}_{-0.039}$ | $0.573^{+0.028}_{-0.045}$ | $-0.414^{+0.055}_{-0.078}$ | $2.630^{+0.122}_{-0.198}$ | $0.185^{+0.029}_{-0.035}$ |
| New | $0.668^{+0.060}_{-0.091}$ | $-0.347^{+0.018}_{-0.029}$ | $0.363^{+0.022}_{-0.035}$ | $-0.128^{+0.052}_{-0.070}$ | $1.307^{+0.084}_{-0.134}$ | $0.176^{+0.033}_{-0.041}$ |

TABLE XV: The LECs come from sum rules in [14]. They are in units of 10^{-3} GeV^{-2} .

| | $C_{12} + C_{61} + C_{80}$ | $C_{12} - C_{61} + C_{80}$ | C_{61} | $C_{12} + C_{80}$ |
|----------------|----------------------------|----------------------------|------------------------|------------------------|
| w_{DK} [14] | 2.48 ± 0.19 | -0.55 ± 0.21 | 1.51 ± 0.19 | 0.97 ± 0.11 |
| \hat{w} [14] | 2.48 ± 0.18 | -0.46 ± 0.19 | 1.47 ± 0.17 | 1.01 ± 0.10 |
| Old[18] | $3.41^{+0.25}_{-0.28}$ | $-2.36^{+0.20}_{-0.24}$ | $2.88^{+0.22}_{-0.26}$ | $0.53^{+0.02}_{-0.02}$ |
| New | $3.09^{+0.22}_{-0.25}$ | $-1.75^{+0.16}_{-0.19}$ | $2.42^{+0.19}_{-0.22}$ | $0.67^{+0.03}_{-0.03}$ |

VI. SUMMARY

In this research, we updated our original LECs to the $O(p^6)$ order, including two and three flavours, and normal and anomalous ones. The new contributions come from $n = 2$ in (1) and $n = 1$ in (4). This is one small step beyond the

TABLE XVI: The obtained values for the p^6 order LECs C_{87}^r . They are in units of 10^{-5} .

| | New | Old[18] | Ref.[57] | Ref.[58] | Ref.[27] |
|------------|------------------------|------------------------|---------------|---------------|-----------------|
| C_{87}^r | $3.63^{+0.22}_{-0.35}$ | $5.73^{+0.28}_{-0.45}$ | 3.1 ± 1.1 | 4.3 ± 0.4 | 3.70 ± 0.14 |

TABLE XVII: Some LECs from different references.

| | $10^5(2C_{63}^r - C_{65}^r)$ | $10^6 C_{38}^r$ | $10^5(C_{88}^r - C_{90}^r)$ | $(C_{14} + C_{15})10^3 \text{GeV}^2$ | $(C_{15} + 2C_{17})10^3 \text{GeV}^2$ |
|---------|------------------------------|------------------------|-----------------------------|--------------------------------------|---------------------------------------|
| | $1.8 \pm 0.7[59]$ | $2 \pm 6[60]$ | $-4.6 \pm 0.4[61]$ | $0.37 \pm 0.08[16]$ | $1.29 \pm 0.16[16]$ |
| | | $8 \pm 5[62]$ | $-4.5 \pm 0.5[63]$ | | |
| Old[18] | $6.36^{+0.48}_{-0.57}$ | $3.08^{+0.62}_{-0.56}$ | $-5.99^{+0.27}_{-0.43}$ | $-0.83^{+0.12}_{-0.19}$ | $0.03^{+0.06}_{-0.02}$ |
| New | $5.89^{+0.45}_{-0.53}$ | $3.57^{+0.34}_{-0.17}$ | $-3.04^{+0.18}_{-0.29}$ | $-0.87^{+0.14}_{-0.21}$ | $0.35^{+0.03}_{-0.08}$ |

TABLE XVIII: The obtained values for the p^6 order LECs from our work. The coefficients in the table are in units of 10^{-3}GeV^{-2} .

| | C_{20} | $-3C_{21}$ | C_{32} | $\frac{1}{6}C_{35}$ | C_{24} | $6C_{28}$ | $3C_{30}$ |
|---------|------------------------|------------------------|------------------------|---------------------------|------------------------|------------------------|------------------------|
| Old[18] | $0.18^{+0.03}_{-0.04}$ | $0.18^{+0.03}_{-0.03}$ | $0.18^{+0.03}_{-0.04}$ | $0.028^{+0.020}_{-0.028}$ | $1.62^{+0.04}_{-0.07}$ | $1.80^{+0.06}_{-0.06}$ | $1.80^{+0.06}_{-0.09}$ |
| New | $0.17^{+0.02}_{-0.03}$ | $0.17^{+0.02}_{-0.03}$ | $0.17^{+0.02}_{-0.03}$ | $0.02^{+0.01}_{-0.02}$ | $0.87^{+0.02}_{-0.04}$ | $1.10^{+0.03}_{-0.05}$ | $1.10^{+0.03}_{-0.05}$ |

GND model. As a check, the $O(p^4)$ -order absolute values have decreased, and are closer to others, so our updates are plausible. Up to the $O(p^6)$ order, the absolute values of the LECs exhibited varying changes or remained unchanged. We also compared these LECs with others. Most of them are much closer than the old values, but some combinations of LECs fare badly. The combinations found in references are directly from phenomenological data, and they can be more precise. However, we have obtained LECs separately. On the whole, the new LECs values are better than old ones; one possible reason for the differences is propagation of errors. In this method, more precise results needs a more detailed analysis of (1), which remains as work for the future.

Acknowledgments

Jiang thanks Professor Rui-Jing Lu for the helpful discussions.. This work was supported by the National Science Foundation of China (NSFC) under Grants No. 11205034 and No. 11475092; the Natural Science Foundation of Guangxi Grants No. 2013GXNSFBA019012 and No. 2013GXNSFFA019001; the Specialized Research Fund for the Doctoral Program of High Education of China No. 20110002110010, and the Tsinghua University Initiative Scientific Research Program No. 20121088494.

Appendix A: The $\Delta\tilde{K}_i$ coefficients

$$\begin{aligned}
\Delta\tilde{K}_1 = & \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{32} \Sigma_k''^2 + \frac{1}{960} \Sigma_k \Sigma_k''' X + \frac{3}{16} \Sigma_k'^4 X - \frac{9}{16} \Sigma_k^2 \Sigma_k''^2 X - \frac{21}{160} \Sigma_k'^2 X^2 + \frac{1}{240} \Sigma_k \Sigma_k'' X^2 \right. \\
& - \frac{277}{480} \Sigma_k \Sigma_k'^3 X^2 - \frac{1}{960} \Sigma_k^3 \Sigma_k''' X^2 - \frac{75}{16} \Sigma_k^2 \Sigma_k'^4 X^2 + \frac{75}{32} \Sigma_k^4 \Sigma_k''^2 X^2 - \frac{7}{320} X^3 - \frac{7}{80} \Sigma_k \Sigma_k' X^3 \\
& \left. + \frac{323}{192} \Sigma_k^2 \Sigma_k'^2 X^3 + \frac{5491}{480} \Sigma_k^3 \Sigma_k'^3 X^3 + \frac{451}{16} \Sigma_k^4 \Sigma_k'^4 X^3 - 4 \Sigma_k^6 \Sigma_k''^2 X^3 + \frac{257}{960} \Sigma_k^2 X^4 - \frac{771}{80} \Sigma_k^4 \Sigma_k'^2 X^4 \right]
\end{aligned}$$

$$\begin{aligned}
& -\frac{13087}{240}\Sigma_k^5\Sigma_k'^3X^4 - \frac{3467}{48}\Sigma_k^6\Sigma_k'^4X^4 + \frac{49}{16}\Sigma_k^8\Sigma_k''^2X^4 - \frac{11}{12}\Sigma_k^4X^5 + \frac{139}{6}\Sigma_k^6\Sigma_k'^2X^5 + \frac{1259}{12}\Sigma_k^7\Sigma_k'^3X^5 \\
& + \frac{2201}{24}\Sigma_k^8\Sigma_k'^4X^5 - \frac{7}{8}\Sigma_k^{10}\Sigma_k''^2X^5 + \frac{31}{48}\Sigma_k^6X^6 - \frac{70}{3}\Sigma_k^8\Sigma_k'^2X^6 - \frac{357}{4}\Sigma_k^9\Sigma_k'^3X^6 - \frac{343}{6}\Sigma_k^{10}\Sigma_k'^4X^6 \\
& + \frac{9}{16}\Sigma_k^8X^7 + \frac{33}{4}\Sigma_k^{10}\Sigma_k'^2X^7 + 28\Sigma_k^{11}\Sigma_k'^3X^7 + 14\Sigma_k^{12}\Sigma_k'^4X^7 \Big], \\
\Delta\tilde{K}_2 = & \int \frac{d^4k}{(2\pi)^4} \Big[-\frac{1}{96}\Sigma_k''^2 - \frac{31}{432}\Sigma_k'^4X + \frac{23}{144}\Sigma_k^2\Sigma_k''^2X + \frac{1}{48}\Sigma_k'^2X^2 + \frac{11}{48}\Sigma_k\Sigma_k'^3X^2 + \frac{599}{432}\Sigma_k^2\Sigma_k'^4X^2 \\
& - \frac{185}{288}\Sigma_k^4\Sigma_k''^2X^2 - \frac{37}{96}\Sigma_k^2\Sigma_k'^2X^3 - \frac{245}{72}\Sigma_k^3\Sigma_k'^3X^3 - \frac{3391}{432}\Sigma_k^4\Sigma_k'^4X^3 + \frac{13}{12}\Sigma_k^6\Sigma_k''^2X^3 - \frac{1}{24}\Sigma_k^2X^4 \\
& + \frac{245}{96}\Sigma_k^4\Sigma_k'^2X^4 + \frac{2179}{144}\Sigma_k^5\Sigma_k'^3X^4 + \frac{8509}{432}\Sigma_k^6\Sigma_k'^4X^4 - \frac{119}{144}\Sigma_k^8\Sigma_k''^2X^4 + \frac{3}{16}\Sigma_k^4X^5 - \frac{325}{48}\Sigma_k^6\Sigma_k'^2X^5 \\
& - \frac{2051}{72}\Sigma_k^7\Sigma_k'^3X^5 - \frac{5359}{216}\Sigma_k^8\Sigma_k'^4X^5 + \frac{17}{72}\Sigma_k^{10}\Sigma_k''^2X^5 - \frac{3}{16}\Sigma_k^6X^6 + \frac{15}{2}\Sigma_k^8\Sigma_k'^2X^6 + \frac{289}{12}\Sigma_k^9\Sigma_k'^3X^6 \\
& + \frac{833}{54}\Sigma_k^{10}\Sigma_k'^4X^6 + \frac{1}{48}\Sigma_k^8X^7 - \frac{35}{12}\Sigma_k^{10}\Sigma_k'^2X^7 - \frac{68}{9}\Sigma_k^{11}\Sigma_k'^3X^7 - \frac{34}{9}\Sigma_k^{12}\Sigma_k'^4X^7 \Big], \\
\Delta\tilde{K}_3 = & \int \frac{d^4k}{(2\pi)^4} \Big[\frac{1}{48}\Sigma_k''^2 + \frac{1}{960}\Sigma_k\Sigma_k'''X + \frac{25}{216}\Sigma_k'^4X - \frac{29}{72}\Sigma_k^2\Sigma_k''^2X - \frac{7}{120}\Sigma_k'^2X^2 + \frac{1}{240}\Sigma_k\Sigma_k''X^2 \\
& - \frac{167}{480}\Sigma_k\Sigma_k'^3X^2 - \frac{1}{960}\Sigma_k^3\Sigma_k'''X^2 - \frac{713}{216}\Sigma_k^2\Sigma_k'^4X^2 + \frac{245}{144}\Sigma_k^4\Sigma_k''^2X^2 - \frac{7}{320}X^3 - \frac{7}{80}\Sigma_k\Sigma_k'X^3 \\
& + \frac{51}{64}\Sigma_k^2\Sigma_k'^2X^3 + \frac{11573}{1440}\Sigma_k^3\Sigma_k'^3X^3 + \frac{4393}{216}\Sigma_k^4\Sigma_k'^4X^3 - \frac{35}{12}\Sigma_k^6\Sigma_k''^2X^3 + \frac{137}{960}\Sigma_k^2X^4 - \frac{1513}{240}\Sigma_k^4\Sigma_k'^2X^4 \\
& - \frac{14183}{360}\Sigma_k^5\Sigma_k'^3X^4 - \frac{11347}{216}\Sigma_k^6\Sigma_k'^4X^4 + \frac{161}{72}\Sigma_k^8\Sigma_k''^2X^4 - \frac{1}{2}\Sigma_k^4X^5 + \frac{277}{16}\Sigma_k^6\Sigma_k'^2X^5 + \frac{5503}{72}\Sigma_k^7\Sigma_k'^3X^5 \\
& + \frac{7225}{108}\Sigma_k^8\Sigma_k'^4X^5 - \frac{23}{36}\Sigma_k^{10}\Sigma_k''^2X^5 + \frac{9}{16}\Sigma_k^6X^6 - \frac{55}{3}\Sigma_k^8\Sigma_k'^2X^6 - \frac{391}{6}\Sigma_k^9\Sigma_k'^3X^6 - \frac{1127}{27}\Sigma_k^{10}\Sigma_k'^4X^6 \\
& + \frac{13}{48}\Sigma_k^8X^7 + \frac{79}{12}\Sigma_k^{10}\Sigma_k'^2X^7 + \frac{184}{9}\Sigma_k^{11}\Sigma_k'^3X^7 + \frac{92}{9}\Sigma_k^{12}\Sigma_k'^4X^7 \Big], \\
\Delta\tilde{K}_4 = & \int \frac{d^4k}{(2\pi)^4} \Big[\frac{9}{16}\Sigma_k\Sigma_k'^2X^2 - \frac{1}{16}\Sigma_kX^3 - \frac{271}{48}\Sigma_k^3\Sigma_k'^2X^3 + \frac{73}{96}\Sigma_k^3X^4 + \frac{193}{12}\Sigma_k^5\Sigma_k'^2X^4 - \frac{85}{48}\Sigma_k^5X^5 \\
& - \frac{53}{3}\Sigma_k^7\Sigma_k'^2X^5 + \frac{25}{24}\Sigma_k^7X^6 + \frac{20}{3}\Sigma_k^9\Sigma_k'^2X^6 \Big], \\
\Delta\tilde{K}_5 = & \int \frac{d^4k}{(2\pi)^4} \Big[\frac{1}{192}\Sigma_k''^2X + \frac{1}{96}\Sigma_k'X^2 + \frac{35}{96}\Sigma_k\Sigma_k'^2X^2 - \frac{7}{96}\Sigma_kX^3 - \frac{1}{48}\Sigma_k^2\Sigma_k'X^3 - \frac{61}{12}\Sigma_k^3\Sigma_k'^2X^3 \\
& + \frac{19}{24}\Sigma_k^3X^4 + \frac{377}{24}\Sigma_k^5\Sigma_k'^2X^4 - \frac{73}{48}\Sigma_k^5X^5 - \frac{53}{3}\Sigma_k^7\Sigma_k'^2X^5 + \frac{25}{24}\Sigma_k^7X^6 + \frac{20}{3}\Sigma_k^9\Sigma_k'^2X^6 \Big], \\
\Delta\tilde{K}_6 = & \int \frac{d^4k}{(2\pi)^4} \Big[-\frac{1}{192}\Sigma_k''^2X - \frac{1}{96}\Sigma_k'X^2 - \frac{89}{96}\Sigma_k\Sigma_k'^2X^2 + \frac{7}{96}\Sigma_kX^3 + \frac{1}{48}\Sigma_k^2\Sigma_k'X^3 + \frac{515}{48}\Sigma_k^3\Sigma_k'^2X^3 \\
& - \frac{7}{48}\Sigma_k^3X^4 - \frac{763}{24}\Sigma_k^5\Sigma_k'^2X^4 - \frac{49}{48}\Sigma_k^5X^5 + \frac{106}{3}\Sigma_k^7\Sigma_k'^2X^5 + \frac{25}{24}\Sigma_k^7X^6 - \frac{40}{3}\Sigma_k^9\Sigma_k'^2X^6 \Big], \\
\Delta\tilde{K}_7 = & \int \frac{d^4k}{(2\pi)^4} \Big[-\frac{7}{32}\Sigma_k\Sigma_k'^2X^2 + \frac{1}{32}\Sigma_kX^3 + \frac{81}{32}\Sigma_k^3\Sigma_k'^2X^3 - \frac{7}{96}\Sigma_k^3X^4 - \frac{367}{48}\Sigma_k^5\Sigma_k'^2X^4 - \frac{2}{3}\Sigma_k^5X^5 \\
& + \frac{26}{3}\Sigma_k^7\Sigma_k'^2X^5 + \frac{5}{6}\Sigma_k^7X^6 - \frac{10}{3}\Sigma_k^9\Sigma_k'^2X^6 \Big], \\
\Delta\tilde{K}_8 = & \int \frac{d^4k}{(2\pi)^4} \Big[\frac{1}{32}\Sigma_k'^2X - \frac{3}{32}\Sigma_k^2\Sigma_k'^2X^2 + \frac{11}{32}\Sigma_k^2X^3 + \frac{1}{16}\Sigma_k^4\Sigma_k'^2X^3 - \frac{57}{32}\Sigma_k^4X^4 + \frac{3}{2}\Sigma_k^6X^5 \Big], \\
\Delta\tilde{K}_9 = & \int \frac{d^4k}{(2\pi)^4} \Big[\frac{1}{32}\Sigma_k'^2X - \frac{3}{32}\Sigma_k^2\Sigma_k'^2X^2 + \frac{3}{16}\Sigma_k^2X^3 + \frac{1}{16}\Sigma_k^4\Sigma_k'^2X^3 - \frac{15}{16}\Sigma_k^4X^4 + \frac{3}{4}\Sigma_k^6X^5 \Big], \\
\Delta\tilde{K}_{10} = & \int \frac{d^4k}{(2\pi)^4} \Big[\frac{1}{32}\Sigma_kX^2 - \frac{1}{4}\Sigma_k^3X^3 + \frac{1}{4}\Sigma_k^5X^4 \Big],
\end{aligned}$$

$$\begin{aligned}
\Delta \tilde{K}_{11} &= \int \frac{d^4 k}{(2\pi)^4} \left[\frac{1}{48} \Sigma_k'^3 X - \frac{1}{192} \Sigma_k' X^2 + \frac{1}{8} \Sigma_k \Sigma_k'^2 X^2 - \frac{1}{12} \Sigma_k^2 \Sigma_k'^3 X^2 + \frac{7}{192} \Sigma_k X^3 + \frac{1}{48} \Sigma_k^2 \Sigma_k' X^3 \right. \\
&\quad - \frac{7}{12} \Sigma_k^3 \Sigma_k'^2 X^3 + \frac{5}{48} \Sigma_k^4 \Sigma_k'^3 X^3 - \frac{61}{192} \Sigma_k^3 X^4 + \frac{19}{24} \Sigma_k^5 \Sigma_k'^2 X^4 - \frac{1}{24} \Sigma_k^6 \Sigma_k'^3 X^4 + \frac{1}{6} \Sigma_k^5 X^5 \\
&\quad \left. - \frac{1}{3} \Sigma_k^7 \Sigma_k'^2 X^5 \right], \\
\Delta \tilde{K}_{12} &= \int \frac{d^4 k}{(2\pi)^4} \left[\frac{1}{576} \Sigma_k''^2 + \frac{1}{1920} \Sigma_k \Sigma_k''' X + \frac{1}{216} \Sigma_k'^4 X - \frac{7}{144} \Sigma_k^2 \Sigma_k''^2 X + \frac{17}{960} \Sigma_k'^2 X^2 + \frac{1}{480} \Sigma_k \Sigma_k'' X^2 \right. \\
&\quad + \frac{83}{960} \Sigma_k \Sigma_k'^3 X^2 - \frac{1}{1920} \Sigma_k^3 \Sigma_k''' X^2 - \frac{10}{27} \Sigma_k^2 \Sigma_k'^4 X^2 + \frac{125}{576} \Sigma_k^4 \Sigma_k''^2 X^2 - \frac{7}{640} X^3 - \frac{7}{160} \Sigma_k \Sigma_k' X^3 \\
&\quad - \frac{169}{384} \Sigma_k^2 \Sigma_k'^2 X^3 + \frac{491}{960} \Sigma_k^3 \Sigma_k'^3 X^3 + \frac{547}{216} \Sigma_k^4 \Sigma_k'^4 X^3 - \frac{109}{288} \Sigma_k^6 \Sigma_k''^2 X^3 - \frac{43}{1920} \Sigma_k^2 X^4 + \frac{203}{320} \Sigma_k^4 \Sigma_k'^2 X^4 \\
&\quad - \frac{6421}{1440} \Sigma_k^5 \Sigma_k'^3 X^4 - \frac{27}{4} \Sigma_k^6 \Sigma_k'^4 X^4 + \frac{7}{24} \Sigma_k^8 \Sigma_k''^2 X^4 + \frac{25}{96} \Sigma_k^4 X^5 + \frac{35}{24} \Sigma_k^6 \Sigma_k'^2 X^5 + \frac{349}{36} \Sigma_k^7 \Sigma_k'^3 X^5 \\
&\quad + \frac{313}{36} \Sigma_k^8 \Sigma_k'^4 X^5 - \frac{1}{12} \Sigma_k^{10} \Sigma_k''^2 X^5 + \frac{1}{24} \Sigma_k^6 X^6 - \frac{35}{12} \Sigma_k^8 \Sigma_k'^2 X^6 - \frac{17}{2} \Sigma_k^9 \Sigma_k'^3 X^6 - \frac{49}{9} \Sigma_k^{10} \Sigma_k'^4 X^6 \\
&\quad - \frac{1}{16} \Sigma_k^8 X^7 + \frac{5}{4} \Sigma_k^{10} \Sigma_k'^2 X^7 + \frac{8}{3} \Sigma_k^{11} \Sigma_k'^3 X^7 + \frac{4}{3} \Sigma_k^{12} \Sigma_k'^4 X^7 \left. \right], \\
\Delta \tilde{K}_{13} &= \int \frac{d^4 k}{(2\pi)^4} \left[-\frac{1}{24} \Sigma_k'^3 X + \frac{1}{96} \Sigma_k' X^2 - \frac{1}{4} \Sigma_k \Sigma_k'^2 X^2 + \frac{1}{6} \Sigma_k^2 \Sigma_k'^3 X^2 + \frac{5}{96} \Sigma_k X^3 - \frac{1}{24} \Sigma_k^2 \Sigma_k' X^3 \right. \\
&\quad + \frac{7}{6} \Sigma_k^3 \Sigma_k'^2 X^3 - \frac{5}{24} \Sigma_k^4 \Sigma_k'^3 X^3 - \frac{59}{96} \Sigma_k^3 X^4 - \frac{19}{12} \Sigma_k^5 \Sigma_k'^2 X^4 + \frac{1}{12} \Sigma_k^6 \Sigma_k'^3 X^4 + \frac{11}{12} \Sigma_k^5 X^5 \\
&\quad \left. + \frac{2}{3} \Sigma_k^7 \Sigma_k'^2 X^5 \right], \\
\Delta \tilde{K}_{14} &= \int \frac{d^4 k}{(2\pi)^4} \left[-\frac{13}{288} \Sigma_k''^2 - \frac{1}{320} \Sigma_k \Sigma_k''' X - \frac{13}{54} \Sigma_k'^4 X + \frac{65}{72} \Sigma_k^2 \Sigma_k''^2 X + \frac{49}{480} \Sigma_k'^2 X^2 - \frac{1}{80} \Sigma_k \Sigma_k'' X^2 \right. \\
&\quad + \frac{251}{480} \Sigma_k \Sigma_k'^3 X^2 + \frac{1}{320} \Sigma_k^3 \Sigma_k''' X^2 + \frac{793}{108} \Sigma_k^2 \Sigma_k'^4 X^2 - \frac{1105}{288} \Sigma_k^4 \Sigma_k''^2 X^2 + \frac{21}{320} X^3 + \frac{21}{80} \Sigma_k \Sigma_k' X^3 \\
&\quad - \frac{145}{192} \Sigma_k^2 \Sigma_k'^2 X^3 - \frac{24619}{1440} \Sigma_k^3 \Sigma_k'^3 X^3 - \frac{1235}{27} \Sigma_k^4 \Sigma_k'^4 X^3 + \frac{949}{144} \Sigma_k^6 \Sigma_k''^2 X^3 - \frac{311}{960} \Sigma_k^2 X^4 + \frac{4213}{480} \Sigma_k^4 \Sigma_k'^2 X^4 \\
&\quad + \frac{7017}{80} \Sigma_k^5 \Sigma_k'^3 X^4 + \frac{12805}{108} \Sigma_k^6 \Sigma_k'^4 X^4 - \frac{91}{18} \Sigma_k^8 \Sigma_k''^2 X^4 + \frac{37}{48} \Sigma_k^4 X^5 - \frac{217}{8} \Sigma_k^6 \Sigma_k'^2 X^5 - \frac{689}{4} \Sigma_k^7 \Sigma_k'^3 X^5 \\
&\quad - \frac{4082}{27} \Sigma_k^8 \Sigma_k'^4 X^5 + \frac{13}{9} \Sigma_k^{10} \Sigma_k''^2 X^5 - \frac{1}{6} \Sigma_k^6 X^6 + \frac{175}{6} \Sigma_k^8 \Sigma_k'^2 X^6 + \frac{442}{3} \Sigma_k^9 \Sigma_k'^3 X^6 + \frac{2548}{27} \Sigma_k^{10} \Sigma_k'^4 X^6 \\
&\quad - \frac{43}{24} \Sigma_k^8 X^7 - \frac{61}{6} \Sigma_k^{10} \Sigma_k'^2 X^7 - \frac{416}{9} \Sigma_k^{11} \Sigma_k'^3 X^7 - \frac{208}{9} \Sigma_k^{12} \Sigma_k'^4 X^7 \left. \right], \\
\Delta \tilde{K}_{15} &= \int \frac{d^4 k}{(2\pi)^4} \left[\frac{1}{32} \Sigma_k'^2 X + \frac{13}{32} \Sigma_k^2 \Sigma_k'^2 X^2 + \frac{3}{32} \Sigma_k^2 X^3 - \frac{31}{16} \Sigma_k^4 \Sigma_k'^2 X^3 - \frac{13}{32} \Sigma_k^4 X^4 + \frac{5}{2} \Sigma_k^6 \Sigma_k'^2 X^4 \right. \\
&\quad \left. + \frac{1}{4} \Sigma_k^6 X^5 - \Sigma_k^8 \Sigma_k'^2 X^5 \right], \\
\Delta \tilde{K}_{16} &= \int \frac{d^4 k}{(2\pi)^4} \left[-\frac{5}{96} \Sigma_k'^3 X + \frac{1}{192} \Sigma_k' X^2 + \frac{1}{16} \Sigma_k \Sigma_k'^2 X^2 + \frac{5}{24} \Sigma_k^2 \Sigma_k'^3 X^2 - \frac{1}{192} \Sigma_k X^3 - \frac{1}{48} \Sigma_k^2 \Sigma_k' X^3 \right. \\
&\quad - \frac{59}{48} \Sigma_k^3 \Sigma_k'^2 X^3 - \frac{25}{96} \Sigma_k^4 \Sigma_k'^3 X^3 - \frac{35}{192} \Sigma_k^3 X^4 + \frac{55}{12} \Sigma_k^5 \Sigma_k'^2 X^4 + \frac{5}{48} \Sigma_k^6 \Sigma_k'^3 X^4 + \frac{43}{48} \Sigma_k^5 X^5 \\
&\quad \left. - \frac{71}{12} \Sigma_k^7 \Sigma_k'^2 X^5 - \frac{5}{8} \Sigma_k^7 X^6 + \frac{5}{2} \Sigma_k^9 \Sigma_k'^2 X^6 \right], \\
\Delta \tilde{K}_{17} &= \int \frac{d^4 k}{(2\pi)^4} \left[-\frac{5}{144} \Sigma_k''^2 - \frac{1}{480} \Sigma_k \Sigma_k''' X - \frac{85}{432} \Sigma_k'^4 X + \frac{95}{144} \Sigma_k^2 \Sigma_k''^2 X + \frac{31}{480} \Sigma_k'^2 X^2 - \frac{1}{120} \Sigma_k \Sigma_k'' X^2 \right. \\
&\quad + \frac{97}{240} \Sigma_k \Sigma_k'^3 X^2 + \frac{1}{480} \Sigma_k^3 \Sigma_k''' X^2 + \frac{2345}{432} \Sigma_k^2 \Sigma_k'^4 X^2 - \frac{25}{9} \Sigma_k^4 \Sigma_k''^2 X^2 + \frac{7}{160} X^3 + \frac{7}{40} \Sigma_k \Sigma_k' X^3 \\
&\quad - \frac{5}{96} \Sigma_k^2 \Sigma_k'^2 X^3 - \frac{997}{80} \Sigma_k^3 \Sigma_k'^3 X^3 - \frac{14365}{432} \Sigma_k^4 \Sigma_k'^4 X^3 + \frac{685}{144} \Sigma_k^6 \Sigma_k''^2 X^3 - \frac{107}{480} \Sigma_k^2 X^4 + \frac{121}{40} \Sigma_k^4 \Sigma_k'^2 X^4 \\
&\quad \left. - \frac{1}{12} \Sigma_k^6 \Sigma_k'^2 X^4 + \frac{1}{24} \Sigma_k^8 \Sigma_k'^2 X^4 - \frac{1}{24} \Sigma_k^6 X^5 + \frac{1}{24} \Sigma_k^8 \Sigma_k'^2 X^5 - \frac{1}{24} \Sigma_k^6 X^6 + \frac{1}{24} \Sigma_k^8 \Sigma_k'^2 X^6 \right. \\
&\quad \left. - \frac{1}{24} \Sigma_k^6 X^7 + \frac{1}{24} \Sigma_k^8 \Sigma_k'^2 X^7 - \frac{1}{24} \Sigma_k^6 X^8 + \frac{1}{24} \Sigma_k^8 \Sigma_k'^2 X^8 \right].
\end{aligned}$$

$$\begin{aligned}
& + \frac{22841}{360} \Sigma_k^5 \Sigma_k'^3 X^4 + \frac{4115}{48} \Sigma_k^6 \Sigma_k'^4 X^4 - \frac{175}{48} \Sigma_k^8 \Sigma_k''^2 X^4 + \frac{23}{96} \Sigma_k^4 X^5 - \frac{145}{12} \Sigma_k^6 \Sigma_k'^2 X^5 - \frac{4475}{36} \Sigma_k^7 \Sigma_k'^3 X^5 \\
& - \frac{7855}{72} \Sigma_k^8 \Sigma_k'^4 X^5 + \frac{25}{24} \Sigma_k^{10} \Sigma_k''^2 X^5 + \frac{65}{96} \Sigma_k^6 X^6 + \frac{85}{6} \Sigma_k^8 \Sigma_k'^2 X^6 + \frac{425}{4} \Sigma_k^9 \Sigma_k'^3 X^6 + \frac{1225}{18} \Sigma_k^{10} \Sigma_k'^4 X^6 \\
& - \frac{59}{32} \Sigma_k^8 X^7 - \frac{41}{8} \Sigma_k^{10} \Sigma_k'^2 X^7 - \frac{100}{3} \Sigma_k^{11} \Sigma_k'^3 X^7 - \frac{50}{3} \Sigma_k^{12} \Sigma_k'^4 X^7 \Big], \\
\Delta \tilde{K}_{18} &= \int \frac{d^4 k}{(2\pi)^4} \left[\frac{1}{32} \Sigma_k'^2 X + \frac{13}{32} \Sigma_k^2 \Sigma_k'^2 X^2 + \frac{3}{16} \Sigma_k^2 X^3 - \frac{31}{16} \Sigma_k^4 \Sigma_k'^2 X^3 - \frac{11}{16} \Sigma_k^4 X^4 + \frac{5}{2} \Sigma_k^6 \Sigma_k'^2 X^4 \right. \\
& \left. + \frac{1}{4} \Sigma_k^6 X^5 - \Sigma_k^8 \Sigma_k'^2 X^5 \right], \\
\Delta \tilde{K}_{19} &= \int \frac{d^4 k}{(2\pi)^4} \left[-\frac{1}{48} \Sigma_k'^3 X - \frac{1}{96} \Sigma_k' X^2 + \frac{5}{8} \Sigma_k \Sigma_k'^2 X^2 + \frac{1}{12} \Sigma_k^2 \Sigma_k'^3 X^2 - \frac{1}{48} \Sigma_k X^3 + \frac{1}{24} \Sigma_k^2 \Sigma_k' X^3 \right. \\
& - \frac{115}{24} \Sigma_k^3 \Sigma_k'^2 X^3 - \frac{5}{48} \Sigma_k^4 \Sigma_k'^3 X^3 + \frac{1}{48} \Sigma_k^3 X^4 + \frac{37}{3} \Sigma_k^5 \Sigma_k'^2 X^4 + \frac{1}{24} \Sigma_k^6 \Sigma_k'^3 X^4 + \frac{7}{12} \Sigma_k^5 X^5 \\
& \left. - \frac{79}{6} \Sigma_k^7 \Sigma_k'^2 X^5 - \frac{5}{4} \Sigma_k^7 X^6 + 5 \Sigma_k^9 \Sigma_k'^2 X^6 \right], \\
\Delta \tilde{K}_{20} &= \int \frac{d^4 k}{(2\pi)^4} \left[\frac{1}{72} \Sigma_k''^2 + \frac{1}{960} \Sigma_k \Sigma_k''' X + \frac{35}{432} \Sigma_k'^4 X - \frac{37}{144} \Sigma_k^2 \Sigma_k''^2 X - \frac{3}{80} \Sigma_k'^2 X^2 + \frac{1}{240} \Sigma_k \Sigma_k'' X^2 \right. \\
& - \frac{9}{160} \Sigma_k \Sigma_k'^3 X^2 - \frac{1}{960} \Sigma_k^3 \Sigma_k''' X^2 - \frac{919}{432} \Sigma_k^2 \Sigma_k'^4 X^2 + \frac{155}{144} \Sigma_k^4 \Sigma_k''^2 X^2 - \frac{7}{320} X^3 - \frac{7}{80} \Sigma_k \Sigma_k' X^3 \\
& + \frac{7}{64} \Sigma_k^2 \Sigma_k'^2 X^3 + \frac{6373}{1440} \Sigma_k^3 \Sigma_k'^3 X^3 + \frac{5579}{432} \Sigma_k^4 \Sigma_k'^4 X^3 - \frac{265}{144} \Sigma_k^6 \Sigma_k'^2 X^3 + \frac{77}{960} \Sigma_k^2 X^4 - \frac{1141}{480} \Sigma_k^4 \Sigma_k'^2 X^4 \\
& - \frac{481}{20} \Sigma_k^5 \Sigma_k'^3 X^4 - \frac{14341}{432} \Sigma_k^6 \Sigma_k'^4 X^4 + \frac{203}{144} \Sigma_k^8 \Sigma_k''^2 X^4 - \frac{1}{48} \Sigma_k^4 X^5 + \frac{415}{48} \Sigma_k^6 \Sigma_k'^2 X^5 + \frac{383}{8} \Sigma_k^7 \Sigma_k'^3 X^5 \\
& + \frac{9115}{216} \Sigma_k^8 \Sigma_k'^4 X^5 - \frac{29}{72} \Sigma_k^{10} \Sigma_k''^2 X^5 + \frac{1}{6} \Sigma_k^6 X^6 - 10 \Sigma_k^8 \Sigma_k'^2 X^6 - \frac{493}{12} \Sigma_k^9 \Sigma_k'^3 X^6 - \frac{1421}{54} \Sigma_k^{10} \Sigma_k'^4 X^6 \\
& \left. + \frac{7}{24} \Sigma_k^8 X^7 + \frac{11}{3} \Sigma_k^{10} \Sigma_k'^2 X^7 + \frac{116}{9} \Sigma_k^{11} \Sigma_k'^3 X^7 + \frac{58}{9} \Sigma_k^{12} \Sigma_k'^4 X^7 \right], \\
\Delta \tilde{K}_{21} &= \int \frac{d^4 k}{(2\pi)^4} \left[-\frac{1}{32} \Sigma_k X^2 + \frac{1}{8} \Sigma_k^3 X^3 \right], \\
\Delta \tilde{K}_{22} &= \int \frac{d^4 k}{(2\pi)^4} \left[-\frac{1}{32} \Sigma_k'^2 X + \frac{3}{32} \Sigma_k^2 \Sigma_k'^2 X^2 - \frac{3}{32} \Sigma_k^2 X^3 - \frac{1}{16} \Sigma_k^4 \Sigma_k'^2 X^3 + \frac{9}{32} \Sigma_k^4 X^4 \right], \\
\Delta \tilde{K}_{23} &= \int \frac{d^4 k}{(2\pi)^4} \left[-\frac{1}{16} \Sigma_k'^3 X + \frac{1}{2} \Sigma_k \Sigma_k'^2 X^2 + \frac{1}{4} \Sigma_k^2 \Sigma_k'^3 X^2 - \frac{1}{32} \Sigma_k X^3 - \frac{37}{8} \Sigma_k^3 \Sigma_k'^2 X^3 - \frac{5}{16} \Sigma_k^4 \Sigma_k'^3 X^3 \right. \\
& - \frac{1}{48} \Sigma_k^3 X^4 + \frac{317}{24} \Sigma_k^5 \Sigma_k'^2 X^4 + \frac{1}{8} \Sigma_k^6 \Sigma_k'^3 X^4 + \frac{53}{48} \Sigma_k^5 X^5 - \frac{179}{12} \Sigma_k^7 \Sigma_k'^2 X^5 - \frac{35}{24} \Sigma_k^7 X^6 \\
& \left. + \frac{35}{6} \Sigma_k^9 \Sigma_k'^2 X^6 \right], \\
\Delta \tilde{K}_{24} &= \int \frac{d^4 k}{(2\pi)^4} \left[-\Sigma_k^2 \Sigma_k'^2 X^2 - \frac{3}{16} \Sigma_k^2 X^3 + 4 \Sigma_k^4 \Sigma_k'^2 X^3 + \frac{13}{16} \Sigma_k^4 X^4 - 5 \Sigma_k^6 \Sigma_k'^2 X^4 - \frac{1}{2} \Sigma_k^6 X^5 \right. \\
& \left. + 2 \Sigma_k^8 \Sigma_k'^2 X^5 \right], \\
\Delta \tilde{K}_{25} &= \int \frac{d^4 k}{(2\pi)^4} \left[-\frac{1}{16} \Sigma_k'^3 X + \frac{3}{8} \Sigma_k \Sigma_k'^2 X^2 + \frac{1}{4} \Sigma_k^2 \Sigma_k'^3 X^2 - \frac{1}{32} \Sigma_k X^3 - \frac{29}{8} \Sigma_k^3 \Sigma_k'^2 X^3 - \frac{5}{16} \Sigma_k^4 \Sigma_k'^3 X^3 \right. \\
& + \frac{3}{32} \Sigma_k^3 X^4 + \frac{43}{4} \Sigma_k^5 \Sigma_k'^2 X^4 + \frac{1}{8} \Sigma_k^6 \Sigma_k'^3 X^4 + \frac{7}{8} \Sigma_k^5 X^5 - \frac{25}{2} \Sigma_k^7 \Sigma_k'^2 X^5 - \frac{5}{4} \Sigma_k^7 X^6 \\
& \left. + 5 \Sigma_k^9 \Sigma_k'^2 X^6 \right], \\
\Delta \tilde{K}_{26} &= \int \frac{d^4 k}{(2\pi)^4} \left[\frac{1}{16} \Sigma_k'^2 X - \frac{19}{16} \Sigma_k^2 \Sigma_k'^2 X^2 + \frac{1}{8} \Sigma_k^2 X^3 + \frac{33}{8} \Sigma_k^4 \Sigma_k'^2 X^3 + \frac{1}{4} \Sigma_k^4 X^4 - 5 \Sigma_k^6 \Sigma_k'^2 X^4 \right.
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{2}\Sigma_k^6 X^5 + 2\Sigma_k^8 \Sigma_k'^2 X^5 \Big], \\
\Delta \tilde{K}_{27} = & \int \frac{d^4 k}{(2\pi)^4} \Big[-\frac{1}{32}\Sigma_k''^2 - \frac{1}{720}\Sigma_k \Sigma_k''' X - \frac{37}{216}\Sigma_k'^4 X + \frac{11}{18}\Sigma_k^2 \Sigma_k''^2 X + \frac{43}{360}\Sigma_k'^2 X^2 - \frac{1}{180}\Sigma_k \Sigma_k'' X^2 \\
& + \frac{49}{120}\Sigma_k \Sigma_k'^3 X^2 + \frac{1}{720}\Sigma_k^3 \Sigma_k''' X^2 + \frac{1079}{216}\Sigma_k^2 \Sigma_k'^4 X^2 - \frac{745}{288}\Sigma_k^4 \Sigma_k''^2 X^2 + \frac{47}{1440}X^3 + \frac{47}{360}\Sigma_k \Sigma_k' X^3 \\
& - \frac{73}{144}\Sigma_k^2 \Sigma_k'^2 X^3 - \frac{2113}{180}\Sigma_k^3 \Sigma_k'^3 X^3 - \frac{6673}{216}\Sigma_k^4 \Sigma_k'^4 X^3 + \frac{71}{16}\Sigma_k^6 \Sigma_k''^2 X^3 - \frac{143}{480}\Sigma_k^2 X^4 + \frac{197}{120}\Sigma_k^4 \Sigma_k'^2 X^4 \\
& + \frac{21359}{360}\Sigma_k^5 \Sigma_k'^3 X^4 + \frac{17257}{216}\Sigma_k^6 \Sigma_k'^4 X^4 - \frac{245}{72}\Sigma_k^8 \Sigma_k''^2 X^4 + \frac{2}{3}\Sigma_k^4 X^5 - \frac{15}{4}\Sigma_k^6 \Sigma_k'^2 X^5 - \frac{2089}{18}\Sigma_k^7 \Sigma_k'^3 X^5 \\
& - \frac{10993}{108}\Sigma_k^8 \Sigma_k'^4 X^5 + \frac{35}{36}\Sigma_k^{10} \Sigma_k''^2 X^5 + \frac{1}{24}\Sigma_k^6 X^6 + \frac{10}{3}\Sigma_k^8 \Sigma_k'^2 X^6 + \frac{595}{6}\Sigma_k^9 \Sigma_k'^3 X^6 + \frac{1715}{27}\Sigma_k^{10} \Sigma_k'^4 X^6 \\
& - \frac{59}{48}\Sigma_k^8 X^7 - \frac{5}{6}\Sigma_k^{10} \Sigma_k'^2 X^7 - \frac{280}{9}\Sigma_k^{11} \Sigma_k'^3 X^7 - \frac{140}{9}\Sigma_k^{12} \Sigma_k'^4 X^7 \Big], \\
\Delta \tilde{K}_{28} = & \int \frac{d^4 k}{(2\pi)^4} \Big[-\frac{1}{48}\Sigma_k''^2 - \frac{1}{960}\Sigma_k \Sigma_k''' X - \frac{13}{108}\Sigma_k'^4 X + \frac{7}{18}\Sigma_k^2 \Sigma_k''^2 X + \frac{13}{480}\Sigma_k'^2 X^2 + \frac{1}{160}\Sigma_k \Sigma_k'' X^2 \\
& + \frac{137}{480}\Sigma_k \Sigma_k'^3 X^2 + \frac{1}{960}\Sigma_k^3 \Sigma_k''' X^2 + \frac{347}{108}\Sigma_k^2 \Sigma_k'^4 X^2 - \frac{235}{144}\Sigma_k^4 \Sigma_k''^2 X^2 + \frac{1}{960}X^3 + \frac{1}{240}\Sigma_k \Sigma_k' X^3 \\
& - \frac{29}{64}\Sigma_k^2 \Sigma_k'^2 X^3 - \frac{10813}{1440}\Sigma_k^3 \Sigma_k'^3 X^3 - \frac{2113}{108}\Sigma_k^4 \Sigma_k'^4 X^3 + \frac{67}{24}\Sigma_k^6 \Sigma_k''^2 X^3 - \frac{127}{960}\Sigma_k^2 X^4 + \frac{931}{480}\Sigma_k^4 \Sigma_k'^2 X^4 \\
& + \frac{26981}{720}\Sigma_k^5 \Sigma_k'^3 X^4 + \frac{5437}{108}\Sigma_k^6 \Sigma_k'^4 X^4 - \frac{77}{36}\Sigma_k^8 \Sigma_k''^2 X^4 + \frac{27}{32}\Sigma_k^4 X^5 - \frac{89}{24}\Sigma_k^6 \Sigma_k'^2 X^5 - \frac{2629}{36}\Sigma_k^7 \Sigma_k'^3 X^5 \\
& - \frac{3457}{54}\Sigma_k^8 \Sigma_k'^4 X^5 + \frac{11}{18}\Sigma_k^{10} \Sigma_k''^2 X^5 - \frac{35}{32}\Sigma_k^6 X^6 + \frac{10}{3}\Sigma_k^8 \Sigma_k'^2 X^6 + \frac{187}{3}\Sigma_k^9 \Sigma_k'^3 X^6 + \frac{1078}{27}\Sigma_k^{10} \Sigma_k'^4 X^6 \\
& + \frac{65}{96}\Sigma_k^8 X^7 - \frac{7}{6}\Sigma_k^{10} \Sigma_k'^2 X^7 - \frac{176}{9}\Sigma_k^{11} \Sigma_k'^3 X^7 - \frac{88}{9}\Sigma_k^{12} \Sigma_k'^4 X^7 \Big], \\
\Delta \tilde{K}_{29} = & \int \frac{d^4 k}{(2\pi)^4} \Big[\frac{7}{96}\Sigma_k''^2 + \frac{1}{320}\Sigma_k \Sigma_k''' X + \frac{29}{72}\Sigma_k'^4 X - \frac{17}{12}\Sigma_k^2 \Sigma_k''^2 X - \frac{89}{480}\Sigma_k'^2 X^2 + \frac{1}{480}\Sigma_k \Sigma_k'' X^2 \\
& - \frac{391}{480}\Sigma_k \Sigma_k'^3 X^2 - \frac{1}{320}\Sigma_k^3 \Sigma_k''' X^2 - \frac{835}{72}\Sigma_k^2 \Sigma_k'^4 X^2 + \frac{575}{96}\Sigma_k^4 \Sigma_k''^2 X^2 - \frac{11}{320}X^3 - \frac{11}{80}\Sigma_k \Sigma_k' X^3 \\
& + \frac{265}{192}\Sigma_k^2 \Sigma_k'^2 X^3 + \frac{12793}{480}\Sigma_k^3 \Sigma_k'^3 X^3 + \frac{5153}{72}\Sigma_k^4 \Sigma_k'^4 X^3 - \frac{493}{48}\Sigma_k^6 \Sigma_k''^2 X^3 + \frac{431}{960}\Sigma_k^2 X^4 - \frac{801}{160}\Sigma_k^4 \Sigma_k'^2 X^4 \\
& - \frac{32801}{240}\Sigma_k^5 \Sigma_k'^3 X^4 - \frac{4439}{24}\Sigma_k^6 \Sigma_k'^4 X^4 + \frac{63}{8}\Sigma_k^8 \Sigma_k''^2 X^4 - \frac{7}{6}\Sigma_k^4 X^5 + 9\Sigma_k^6 \Sigma_k'^2 X^5 + \frac{805}{3}\Sigma_k^7 \Sigma_k'^3 X^5 \\
& + \frac{2827}{12}\Sigma_k^8 \Sigma_k'^4 X^5 - \frac{9}{4}\Sigma_k^{10} \Sigma_k''^2 X^5 - \frac{17}{12}\Sigma_k^6 X^6 - \frac{20}{3}\Sigma_k^8 \Sigma_k'^2 X^6 - \frac{459}{2}\Sigma_k^9 \Sigma_k'^3 X^6 - 147\Sigma_k^{10} \Sigma_k'^4 X^6 \\
& + \frac{45}{16}\Sigma_k^8 X^7 + \frac{3}{2}\Sigma_k^{10} \Sigma_k'^2 X^7 + 72\Sigma_k^{11} \Sigma_k'^3 X^7 + 36\Sigma_k^{12} \Sigma_k'^4 X^7 \Big], \\
\Delta \tilde{K}_{30} = & \int \frac{d^4 k}{(2\pi)^4} \Big[\frac{1}{96}\Sigma_k''^2 + \frac{1}{2880}\Sigma_k \Sigma_k''' X + \frac{11}{216}\Sigma_k'^4 X - \frac{2}{9}\Sigma_k^2 \Sigma_k''^2 X - \frac{29}{720}\Sigma_k'^2 X^2 + \frac{1}{720}\Sigma_k \Sigma_k'' X^2 \\
& - \frac{59}{480}\Sigma_k \Sigma_k'^3 X^2 - \frac{1}{2880}\Sigma_k^3 \Sigma_k''' X^2 - \frac{385}{216}\Sigma_k^2 \Sigma_k'^4 X^2 + \frac{275}{288}\Sigma_k^4 \Sigma_k''^2 X^2 + \frac{29}{2880}X^3 + \frac{29}{720}\Sigma_k \Sigma_k' X^3 \\
& + \frac{343}{576}\Sigma_k^2 \Sigma_k'^2 X^3 + \frac{6091}{1440}\Sigma_k^3 \Sigma_k'^3 X^3 + \frac{2447}{216}\Sigma_k^4 \Sigma_k'^4 X^3 - \frac{79}{48}\Sigma_k^6 \Sigma_k''^2 X^3 + \frac{23}{320}\Sigma_k^2 X^4 - \frac{847}{480}\Sigma_k^4 \Sigma_k'^2 X^4 \\
& - \frac{15737}{720}\Sigma_k^5 \Sigma_k'^3 X^4 - \frac{6383}{216}\Sigma_k^6 \Sigma_k'^4 X^4 + \frac{91}{72}\Sigma_k^8 \Sigma_k''^2 X^4 - \frac{37}{96}\Sigma_k^4 X^5 + \frac{37}{24}\Sigma_k^6 \Sigma_k'^2 X^5 + \frac{1549}{36}\Sigma_k^7 \Sigma_k'^3 X^5 \\
& + \frac{4079}{108}\Sigma_k^8 \Sigma_k'^4 X^5 - \frac{13}{36}\Sigma_k^{10} \Sigma_k''^2 X^5 - \frac{61}{96}\Sigma_k^6 X^6 - \frac{221}{6}\Sigma_k^8 \Sigma_k'^2 X^6 - \frac{637}{27}\Sigma_k^{10} \Sigma_k'^4 X^6 + \frac{29}{32}\Sigma_k^8 X^7 \\
& - \frac{1}{3}\Sigma_k^{10} \Sigma_k'^2 X^7 + \frac{104}{9}\Sigma_k^{11} \Sigma_k'^3 X^7 + \frac{52}{9}\Sigma_k^{12} \Sigma_k'^4 X^7 \Big], \\
\Delta \tilde{K}_{31} = & \int \frac{d^4 k}{(2\pi)^4} \Big[-\frac{1}{32}\Sigma_k''^2 - \frac{1}{960}\Sigma_k \Sigma_k''' X - \frac{35}{216}\Sigma_k'^4 X + \frac{23}{36}\Sigma_k^2 \Sigma_k''^2 X + \frac{19}{240}\Sigma_k'^2 X^2 - \frac{1}{240}\Sigma_k \Sigma_k'' X^2
\end{aligned}$$

$$\begin{aligned}
& + \frac{39}{160} \Sigma_k \Sigma_k'^3 X^2 + \frac{1}{960} \Sigma_k^3 \Sigma_k''' X^2 + \frac{1117}{216} \Sigma_k^2 \Sigma_k'^4 X^2 - \frac{785}{288} \Sigma_k^4 \Sigma_k''^2 X^2 - \frac{3}{320} X^3 - \frac{3}{80} \Sigma_k \Sigma_k' X^3 \\
& - \frac{65}{64} \Sigma_k^2 \Sigma_k'^2 X^3 - \frac{16753}{1440} \Sigma_k^3 \Sigma_k'^3 X^3 - \frac{7007}{216} \Sigma_k^4 \Sigma_k'^4 X^3 + \frac{75}{16} \Sigma_k^6 \Sigma_k''^2 X^3 - \frac{49}{320} \Sigma_k^2 X^4 + \frac{1531}{480} \Sigma_k^4 \Sigma_k'^2 X^4 \\
& + \frac{44441}{720} \Sigma_k^5 \Sigma_k'^3 X^4 + \frac{18203}{216} \Sigma_k^6 \Sigma_k'^4 X^4 - \frac{259}{72} \Sigma_k^8 \Sigma_k''^2 X^4 + \frac{73}{96} \Sigma_k^4 X^5 - \frac{37}{12} \Sigma_k^6 \Sigma_k'^2 X^5 - \frac{2201}{18} \Sigma_k^7 \Sigma_k'^3 X^5 \\
& - \frac{11615}{108} \Sigma_k^8 \Sigma_k'^4 X^5 + \frac{37}{36} \Sigma_k^{10} \Sigma_k''^2 X^5 + \frac{133}{96} \Sigma_k^6 X^6 + \frac{629}{6} \Sigma_k^9 \Sigma_k'^3 X^6 + \frac{1813}{27} \Sigma_k^{10} \Sigma_k'^4 X^6 - \frac{199}{96} \Sigma_k^8 X^7 \\
& + \frac{5}{6} \Sigma_k^{10} \Sigma_k'^2 X^7 - \frac{296}{9} \Sigma_k^{11} \Sigma_k'^3 X^7 - \frac{148}{9} \Sigma_k^{12} \Sigma_k'^4 X^7 \Big], \\
\Delta \tilde{K}_{32} = & \int \frac{d^4 k}{(2\pi)^4} \Big[-\frac{1}{32} \Sigma_k''^2 - \frac{1}{960} \Sigma_k \Sigma_k''' X - \frac{85}{432} \Sigma_k'^4 X + \frac{77}{144} \Sigma_k^2 \Sigma_k''^2 X + \frac{23}{480} \Sigma_k'^2 X^2 + \frac{1}{60} \Sigma_k \Sigma_k'' X^2 \\
& + \frac{217}{480} \Sigma_k \Sigma_k'^3 X^2 + \frac{1}{960} \Sigma_k^3 \Sigma_k''' X^2 + \frac{1949}{432} \Sigma_k^2 \Sigma_k'^4 X^2 - \frac{635}{288} \Sigma_k^4 \Sigma_k''^2 X^2 - \frac{19}{960} X^3 - \frac{19}{240} \Sigma_k \Sigma_k' X^3 \\
& - \frac{109}{64} \Sigma_k^2 \Sigma_k'^2 X^3 - \frac{14953}{1440} \Sigma_k^3 \Sigma_k'^3 X^3 - \frac{11509}{432} \Sigma_k^4 \Sigma_k'^4 X^3 + \frac{15}{4} \Sigma_k^6 \Sigma_k''^2 X^3 - \frac{77}{960} \Sigma_k^2 X^4 + \frac{2033}{240} \Sigma_k^4 \Sigma_k'^2 X^4 \\
& + \frac{36491}{720} \Sigma_k^5 \Sigma_k'^3 X^4 + \frac{29311}{432} \Sigma_k^6 \Sigma_k'^4 X^4 - \frac{413}{144} \Sigma_k^8 \Sigma_k''^2 X^4 + \frac{1}{48} \Sigma_k^4 X^5 - \frac{139}{8} \Sigma_k^6 \Sigma_k'^2 X^5 - \frac{883}{9} \Sigma_k^7 \Sigma_k'^3 X^5 \\
& - \frac{18565}{216} \Sigma_k^8 \Sigma_k'^4 X^5 + \frac{59}{72} \Sigma_k^{10} \Sigma_k''^2 X^5 + \frac{41}{24} \Sigma_k^6 X^6 + \frac{50}{3} \Sigma_k^8 \Sigma_k'^2 X^6 + \frac{1003}{12} \Sigma_k^9 \Sigma_k'^3 X^6 + \frac{2891}{54} \Sigma_k^{10} \Sigma_k'^4 X^6 \\
& - \frac{11}{12} \Sigma_k^8 X^7 - \frac{37}{6} \Sigma_k^{10} \Sigma_k'^2 X^7 - \frac{236}{9} \Sigma_k^{11} \Sigma_k'^3 X^7 - \frac{118}{9} \Sigma_k^{12} \Sigma_k'^4 X^7 \Big], \\
\Delta \tilde{K}_{33} = & \int \frac{d^4 k}{(2\pi)^4} \Big[\frac{1}{288} \Sigma_k''^2 + \frac{1}{480} \Sigma_k \Sigma_k''' X + \frac{7}{108} \Sigma_k'^4 X + \frac{5}{72} \Sigma_k^2 \Sigma_k''^2 X - \frac{1}{30} \Sigma_k'^2 X^2 + \frac{1}{120} \Sigma_k \Sigma_k'' X^2 \\
& + \frac{1}{80} \Sigma_k \Sigma_k'^3 X^2 - \frac{1}{480} \Sigma_k^3 \Sigma_k''' X^2 + \frac{17}{54} \Sigma_k^2 \Sigma_k'^4 X^2 - \frac{115}{288} \Sigma_k^4 \Sigma_k''^2 X^2 - \frac{41}{480} X^3 - \frac{41}{120} \Sigma_k \Sigma_k' X^3 \\
& - \frac{39}{32} \Sigma_k^2 \Sigma_k'^2 X^3 - \frac{133}{80} \Sigma_k^3 \Sigma_k'^3 X^3 - \frac{455}{108} \Sigma_k^4 \Sigma_k'^4 X^3 + \frac{107}{144} \Sigma_k^6 \Sigma_k''^2 X^3 + \frac{17}{480} \Sigma_k^2 X^4 - \frac{177}{80} \Sigma_k^4 \Sigma_k'^2 X^4 \\
& + \frac{1727}{180} \Sigma_k^5 \Sigma_k'^3 X^4 + \frac{115}{9} \Sigma_k^6 \Sigma_k'^4 X^4 - \frac{7}{12} \Sigma_k^8 \Sigma_k''^2 X^4 + \frac{25}{24} \Sigma_k^4 X^5 + \frac{137}{6} \Sigma_k^6 \Sigma_k'^2 X^5 - \frac{353}{18} \Sigma_k^7 \Sigma_k'^3 X^5 \\
& - \frac{103}{6} \Sigma_k^8 \Sigma_k'^4 X^5 + \frac{1}{6} \Sigma_k^{10} \Sigma_k''^2 X^5 + \frac{11}{3} \Sigma_k^6 X^6 - \frac{100}{3} \Sigma_k^8 \Sigma_k'^2 X^6 + 17 \Sigma_k^9 \Sigma_k'^3 X^6 + \frac{98}{9} \Sigma_k^{10} \Sigma_k'^4 X^6 \\
& - 4 \Sigma_k^8 X^7 + 14 \Sigma_k^{10} \Sigma_k'^2 X^7 - \frac{16}{3} \Sigma_k^{11} \Sigma_k'^3 X^7 - \frac{8}{3} \Sigma_k^{12} \Sigma_k'^4 X^7 \Big], \\
\Delta \tilde{K}_{34} = & \int \frac{d^4 k}{(2\pi)^4} \Big[-\frac{17}{288} \Sigma_k''^2 - \frac{71}{216} \Sigma_k'^4 X + \frac{41}{36} \Sigma_k^2 \Sigma_k''^2 X + \frac{13}{48} \Sigma_k'^2 X^2 + \frac{11}{12} \Sigma_k \Sigma_k'^3 X^2 + \frac{2017}{216} \Sigma_k^2 \Sigma_k'^4 X^2 \\
& - \frac{1385}{288} \Sigma_k^4 \Sigma_k''^2 X^2 + \frac{1}{24} X^3 + \frac{1}{6} \Sigma_k \Sigma_k' X^3 - \frac{59}{24} \Sigma_k^2 \Sigma_k'^2 X^3 - \frac{1615}{72} \Sigma_k^3 \Sigma_k'^3 X^3 - \frac{12419}{216} \Sigma_k^4 \Sigma_k'^4 X^3 \\
& + \frac{1187}{144} \Sigma_k^6 \Sigma_k''^2 X^3 - \frac{1}{2} \Sigma_k^2 X^4 + \frac{311}{48} \Sigma_k^4 \Sigma_k'^2 X^4 + \frac{2663}{24} \Sigma_k^5 \Sigma_k'^3 X^4 + \frac{32071}{216} \Sigma_k^6 \Sigma_k'^4 X^4 - \frac{455}{72} \Sigma_k^8 \Sigma_k''^2 X^4 \\
& + \frac{1}{3} \Sigma_k^4 X^5 - \frac{71}{12} \Sigma_k^6 \Sigma_k'^2 X^5 - \frac{1295}{6} \Sigma_k^7 \Sigma_k'^3 X^5 - \frac{20419}{108} \Sigma_k^8 \Sigma_k'^4 X^5 + \frac{65}{36} \Sigma_k^{10} \Sigma_k''^2 X^5 + \frac{55}{12} \Sigma_k^6 X^6 \\
& + \frac{1105}{6} \Sigma_k^9 \Sigma_k'^3 X^6 + \frac{3185}{27} \Sigma_k^{10} \Sigma_k'^4 X^6 - \frac{35}{6} \Sigma_k^8 X^7 + \frac{5}{3} \Sigma_k^{10} \Sigma_k'^2 X^7 - \frac{520}{9} \Sigma_k^{11} \Sigma_k'^3 X^7 - \frac{260}{9} \Sigma_k^{12} \Sigma_k'^4 X^7 \\
& \Big], \\
\Delta \tilde{K}_{35} = & \int \frac{d^4 k}{(2\pi)^4} \Big[\frac{5}{288} \Sigma_k''^2 - \frac{1}{480} \Sigma_k \Sigma_k''' X + \frac{7}{72} \Sigma_k'^4 X - \frac{1}{3} \Sigma_k^2 \Sigma_k''^2 X + \frac{1}{30} \Sigma_k'^2 X^2 - \frac{1}{120} \Sigma_k \Sigma_k'' X^2 \\
& - \frac{53}{240} \Sigma_k \Sigma_k'^3 X^2 + \frac{1}{480} \Sigma_k^3 \Sigma_k''' X^2 - \frac{197}{72} \Sigma_k^2 \Sigma_k'^4 X^2 + \frac{45}{32} \Sigma_k^4 \Sigma_k''^2 X^2 + \frac{7}{160} X^3 + \frac{7}{40} \Sigma_k \Sigma_k' X^3 \\
& + \frac{143}{96} \Sigma_k^2 \Sigma_k'^2 X^3 + \frac{4577}{720} \Sigma_k^3 \Sigma_k'^3 X^3 + \frac{1211}{72} \Sigma_k^4 \Sigma_k'^4 X^3 - \frac{347}{144} \Sigma_k^6 \Sigma_k''^2 X^3 - \frac{37}{480} \Sigma_k^2 X^4 - \frac{133}{30} \Sigma_k^4 \Sigma_k'^2 X^4 \\
& \Big]
\end{aligned}$$

$$\begin{aligned}
& -\frac{11579}{360}\Sigma_k^5\Sigma_k'^3X^4 - \frac{9377}{216}\Sigma_k^6\Sigma_k'^4X^4 + \frac{133}{72}\Sigma_k^8\Sigma_k''^2X^4 - \frac{1}{2}\Sigma_k^4X^5 + \frac{1}{24}\Sigma_k^6\Sigma_k'^2X^5 + \frac{2267}{36}\Sigma_k^7\Sigma_k'^3X^5 \\
& + \frac{5969}{108}\Sigma_k^8\Sigma_k'^4X^5 - \frac{19}{36}\Sigma_k^{10}\Sigma_k''^2X^5 - \frac{19}{8}\Sigma_k^6X^6 + \frac{20}{3}\Sigma_k^8\Sigma_k'^2X^6 - \frac{323}{6}\Sigma_k^9\Sigma_k'^3X^6 - \frac{931}{27}\Sigma_k^{10}\Sigma_k'^4X^6 \\
& + \frac{61}{24}\Sigma_k^8X^7 - \frac{23}{6}\Sigma_k^{10}\Sigma_k'^2X^7 + \frac{152}{9}\Sigma_k^{11}\Sigma_k'^3X^7 + \frac{76}{9}\Sigma_k^{12}\Sigma_k'^4X^7 \Big], \\
\Delta\tilde{K}_{36} = & \int \frac{d^4k}{(2\pi)^4} \Big[-\frac{5}{288}\Sigma_k''^2 - \frac{1}{480}\Sigma_k\Sigma_k'''X - \frac{1}{12}\Sigma_k'^4X + \frac{3}{8}\Sigma_k^2\Sigma_k''^2X + \frac{21}{80}\Sigma_k'^2X^2 + \frac{1}{80}\Sigma_k\Sigma_k''X^2 \\
& + \frac{37}{240}\Sigma_k\Sigma_k'^3X^2 + \frac{1}{480}\Sigma_k^3\Sigma_k'''X^2 + 3\Sigma_k^2\Sigma_k'^4X^2 - \frac{155}{96}\Sigma_k^4\Sigma_k''^2X^2 - \frac{3}{160}X^3 - \frac{3}{40}\Sigma_k\Sigma_k'X^3 \\
& - \frac{367}{96}\Sigma_k^2\Sigma_k'^2X^3 - \frac{5003}{720}\Sigma_k^3\Sigma_k'^3X^3 - \frac{689}{36}\Sigma_k^4\Sigma_k'^4X^3 + \frac{401}{144}\Sigma_k^6\Sigma_k''^2X^3 - \frac{127}{480}\Sigma_k^2X^4 + \frac{1109}{60}\Sigma_k^4\Sigma_k'^2X^4 \\
& + \frac{6613}{180}\Sigma_k^5\Sigma_k'^3X^4 + \frac{2699}{54}\Sigma_k^6\Sigma_k'^4X^4 - \frac{77}{36}\Sigma_k^8\Sigma_k''^2X^4 + \frac{41}{24}\Sigma_k^4X^5 - \frac{121}{3}\Sigma_k^6\Sigma_k'^2X^5 - \frac{1309}{18}\Sigma_k^7\Sigma_k'^3X^5 \\
& - \frac{3451}{54}\Sigma_k^8\Sigma_k'^4X^5 + \frac{11}{18}\Sigma_k^{10}\Sigma_k''^2X^5 - \frac{8}{3}\Sigma_k^6X^6 + 40\Sigma_k^8\Sigma_k'^2X^6 + \frac{187}{3}\Sigma_k^9\Sigma_k'^3X^6 + \frac{1078}{27}\Sigma_k^{10}\Sigma_k'^4X^6 \\
& + \frac{11}{6}\Sigma_k^8X^7 - \frac{44}{3}\Sigma_k^{10}\Sigma_k'^2X^7 - \frac{176}{9}\Sigma_k^{11}\Sigma_k'^3X^7 - \frac{88}{9}\Sigma_k^{12}\Sigma_k'^4X^7 \Big], \\
\Delta\tilde{K}_{37} = & \int \frac{d^4k}{(2\pi)^4} \Big[\frac{5}{288}\Sigma_k''^2 + \frac{1}{480}\Sigma_k\Sigma_k'''X + \frac{1}{12}\Sigma_k'^4X - \frac{3}{8}\Sigma_k^2\Sigma_k''^2X - \frac{19}{120}\Sigma_k'^2X^2 + \frac{1}{120}\Sigma_k\Sigma_k''X^2 \\
& - \frac{37}{240}\Sigma_k\Sigma_k'^3X^2 - \frac{1}{480}\Sigma_k^3\Sigma_k'''X^2 - 3\Sigma_k^2\Sigma_k'^4X^2 + \frac{155}{96}\Sigma_k^4\Sigma_k''^2X^2 - \frac{11}{480}X^3 - \frac{11}{120}\Sigma_k\Sigma_k'X^3 \\
& + \frac{191}{96}\Sigma_k^2\Sigma_k'^2X^3 + \frac{5003}{720}\Sigma_k^3\Sigma_k'^3X^3 + \frac{689}{36}\Sigma_k^4\Sigma_k'^4X^3 - \frac{401}{144}\Sigma_k^6\Sigma_k''^2X^3 + \frac{37}{480}\Sigma_k^2X^4 - \frac{839}{60}\Sigma_k^4\Sigma_k'^2X^4 \\
& - \frac{6613}{180}\Sigma_k^5\Sigma_k'^3X^4 - \frac{2699}{54}\Sigma_k^6\Sigma_k'^4X^4 + \frac{77}{36}\Sigma_k^8\Sigma_k''^2X^4 - \frac{7}{12}\Sigma_k^4X^5 + \frac{75}{2}\Sigma_k^6\Sigma_k'^2X^5 + \frac{1309}{18}\Sigma_k^7\Sigma_k'^3X^5 \\
& + \frac{3451}{54}\Sigma_k^8\Sigma_k'^4X^5 - \frac{11}{18}\Sigma_k^{10}\Sigma_k''^2X^5 + \frac{8}{3}\Sigma_k^6X^6 - 40\Sigma_k^8\Sigma_k'^2X^6 - \frac{187}{3}\Sigma_k^9\Sigma_k'^3X^6 - \frac{1078}{27}\Sigma_k^{10}\Sigma_k'^4X^6 \\
& - \frac{11}{6}\Sigma_k^8X^7 + \frac{44}{3}\Sigma_k^{10}\Sigma_k'^2X^7 + \frac{176}{9}\Sigma_k^{11}\Sigma_k'^3X^7 + \frac{88}{9}\Sigma_k^{12}\Sigma_k'^4X^7 \Big], \\
\Delta\tilde{K}_{38} = & \int \frac{d^4k}{(2\pi)^4} \Big[-\frac{1}{144}\Sigma_k''^2 - \frac{1}{240}\Sigma_k\Sigma_k'''X - \frac{5}{54}\Sigma_k'^4X - \frac{1}{36}\Sigma_k^2\Sigma_k''^2X + \frac{1}{15}\Sigma_k'^2X^2 - \frac{1}{60}\Sigma_k\Sigma_k''X^2 \\
& - \frac{1}{40}\Sigma_k\Sigma_k'^3X^2 + \frac{1}{240}\Sigma_k^3\Sigma_k'''X^2 + \frac{2}{27}\Sigma_k^2\Sigma_k'^4X^2 + \frac{35}{144}\Sigma_k^4\Sigma_k''^2X^2 + \frac{7}{80}X^3 + \frac{7}{20}\Sigma_k\Sigma_k'X^3 \\
& + \frac{173}{48}\Sigma_k^2\Sigma_k'^2X^3 + \frac{397}{360}\Sigma_k^3\Sigma_k'^3X^3 + \frac{121}{54}\Sigma_k^4\Sigma_k'^4X^3 - \frac{35}{72}\Sigma_k^6\Sigma_k''^2X^3 - \frac{77}{240}\Sigma_k^2X^4 - \frac{1969}{120}\Sigma_k^4\Sigma_k'^2X^4 \\
& - \frac{63}{10}\Sigma_k^5\Sigma_k'^3X^4 - \frac{217}{27}\Sigma_k^6\Sigma_k'^4X^4 + \frac{7}{18}\Sigma_k^8\Sigma_k''^2X^4 - \frac{9}{4}\Sigma_k^4X^5 + \frac{70}{3}\Sigma_k^6\Sigma_k'^2X^5 + 13\Sigma_k^7\Sigma_k'^3X^5 \\
& + \frac{305}{27}\Sigma_k^8\Sigma_k'^4X^5 - \frac{1}{9}\Sigma_k^{10}\Sigma_k''^2X^5 + 2\Sigma_k^6X^6 - \frac{40}{3}\Sigma_k^8\Sigma_k'^2X^6 - \frac{34}{3}\Sigma_k^9\Sigma_k'^3X^6 - \frac{196}{27}\Sigma_k^{10}\Sigma_k'^4X^6 \\
& - \frac{1}{3}\Sigma_k^8X^7 + \frac{8}{3}\Sigma_k^{10}\Sigma_k'^2X^7 + \frac{32}{9}\Sigma_k^{11}\Sigma_k'^3X^7 + \frac{16}{9}\Sigma_k^{12}\Sigma_k'^4X^7 \Big], \\
\Delta\tilde{K}_{39} = & \int \frac{d^4k}{(2\pi)^4} \Big[\frac{1}{120}\Sigma_k\Sigma_k'''X - \frac{5}{108}\Sigma_k'^4X - \frac{5}{36}\Sigma_k^2\Sigma_k''^2X - \frac{11}{120}\Sigma_k'^2X^2 + \frac{1}{30}\Sigma_k\Sigma_k''X^2 + \frac{23}{60}\Sigma_k\Sigma_k'^3X^2 \\
& - \frac{1}{120}\Sigma_k^3\Sigma_k'''X^2 - \frac{95}{108}\Sigma_k^2\Sigma_k'^4X^2 + \frac{25}{36}\Sigma_k^4\Sigma_k''^2X^2 - \frac{1}{20}X^3 - \frac{1}{5}\Sigma_k\Sigma_k'X^3 - \frac{59}{24}\Sigma_k^2\Sigma_k'^2X^3 \\
& + \frac{233}{180}\Sigma_k^3\Sigma_k'^3X^3 + \frac{835}{108}\Sigma_k^4\Sigma_k'^4X^3 - \frac{5}{4}\Sigma_k^6\Sigma_k''^2X^3 - \frac{13}{120}\Sigma_k^2X^4 + \frac{379}{60}\Sigma_k^4\Sigma_k'^2X^4 - \frac{1291}{90}\Sigma_k^5\Sigma_k'^3X^4 \\
& - \frac{2365}{108}\Sigma_k^6\Sigma_k'^4X^4 + \frac{35}{36}\Sigma_k^8\Sigma_k''^2X^4 + \frac{7}{4}\Sigma_k^4X^5 + 3\Sigma_k^6\Sigma_k'^2X^5 + \frac{289}{9}\Sigma_k^7\Sigma_k'^3X^5 + \frac{1555}{54}\Sigma_k^8\Sigma_k'^4X^5 \\
& - \frac{5}{18}\Sigma_k^{10}\Sigma_k''^2X^5 - \frac{40}{3}\Sigma_k^8\Sigma_k'^2X^6 - \frac{85}{3}\Sigma_k^9\Sigma_k'^3X^6 - \frac{490}{27}\Sigma_k^{10}\Sigma_k'^4X^6 - \frac{5}{6}\Sigma_k^8X^7 + \frac{20}{3}\Sigma_k^{10}\Sigma_k'^2X^7
\end{aligned}$$

$$\begin{aligned}
& + \frac{80}{9} \Sigma_k^{11} \Sigma_k'^3 X^7 + \frac{40}{9} \Sigma_k^{12} \Sigma_k'^4 X^7 \Big], \\
\Delta \tilde{K}_{40} = & \int \frac{d^4 k}{(2\pi)^4} \Big[\frac{1}{288} \Sigma_k''^2 - \frac{1}{480} \Sigma_k \Sigma_k''' X + \frac{5}{72} \Sigma_k'^4 X + \frac{1}{12} \Sigma_k^2 \Sigma_k''^2 X + \frac{53}{240} \Sigma_k'^2 X^2 - \frac{1}{120} \Sigma_k \Sigma_k'' X^2 \\
& - \frac{43}{240} \Sigma_k \Sigma_k'^3 X^2 + \frac{1}{480} \Sigma_k^3 \Sigma_k''' X^2 + \frac{29}{72} \Sigma_k^2 \Sigma_k'^4 X^2 - \frac{15}{32} \Sigma_k^4 \Sigma_k''^2 X^2 + \frac{11}{480} X^3 + \frac{11}{120} \Sigma_k \Sigma_k' X^3 \\
& - \frac{127}{96} \Sigma_k^2 \Sigma_k'^2 X^3 - \frac{863}{720} \Sigma_k^3 \Sigma_k'^3 X^3 - \frac{359}{72} \Sigma_k^4 \Sigma_k'^4 X^3 + \frac{125}{144} \Sigma_k^6 \Sigma_k''^2 X^3 - \frac{17}{480} \Sigma_k^2 X^4 + \frac{1301}{240} \Sigma_k^4 \Sigma_k'^2 X^4 \\
& + \frac{929}{90} \Sigma_k^5 \Sigma_k'^3 X^4 + \frac{3233}{216} \Sigma_k^6 \Sigma_k'^4 X^4 - \frac{49}{72} \Sigma_k^8 \Sigma_k''^2 X^4 - 13 \Sigma_k^6 \Sigma_k'^2 X^5 - \frac{203}{9} \Sigma_k^7 \Sigma_k'^3 X^5 - \frac{2165}{108} \Sigma_k^8 \Sigma_k'^4 X^5 \\
& + \frac{7}{36} \Sigma_k^{10} \Sigma_k''^2 X^5 - \Sigma_k^6 X^6 + \frac{40}{3} \Sigma_k^8 \Sigma_k'^2 X^6 + \frac{119}{6} \Sigma_k^9 \Sigma_k'^3 X^6 + \frac{343}{27} \Sigma_k^{10} \Sigma_k'^4 X^6 + \frac{7}{12} \Sigma_k^8 X^7 \\
& - \frac{14}{3} \Sigma_k^{10} \Sigma_k'^2 X^7 - \frac{56}{9} \Sigma_k^{11} \Sigma_k'^3 X^7 - \frac{28}{9} \Sigma_k^{12} \Sigma_k'^4 X^7 \Big], \\
\Delta \tilde{K}_{41} = & \int \frac{d^4 k}{(2\pi)^4} \Big[\frac{1}{96} \Sigma_k'' X + \frac{1}{48} \Sigma_k' X^2 - \frac{19}{48} \Sigma_k \Sigma_k'^2 X^2 - \frac{1}{12} \Sigma_k X^3 - \frac{1}{24} \Sigma_k^2 \Sigma_k' X^3 + \frac{9}{8} \Sigma_k^3 \Sigma_k'^2 X^3 \\
& + \frac{3}{8} \Sigma_k^3 X^4 - \frac{3}{4} \Sigma_k^5 \Sigma_k'^2 X^4 \Big], \\
\Delta \tilde{K}_{42} = & \int \frac{d^4 k}{(2\pi)^4} \Big[\frac{1}{96} \Sigma_k'' X - \frac{1}{24} \Sigma_k'^3 X + \frac{1}{32} \Sigma_k' X^2 - \frac{1}{12} \Sigma_k \Sigma_k'^2 X^2 + \frac{1}{6} \Sigma_k^2 \Sigma_k'^3 X^2 - \frac{1}{16} \Sigma_k X^3 \\
& - \frac{1}{12} \Sigma_k^2 \Sigma_k' X^3 - \frac{161}{48} \Sigma_k^3 \Sigma_k'^2 X^3 - \frac{5}{24} \Sigma_k^4 \Sigma_k'^3 X^3 - \frac{3}{16} \Sigma_k^3 X^4 + \frac{55}{4} \Sigma_k^5 \Sigma_k'^2 X^4 + \frac{1}{12} \Sigma_k^6 \Sigma_k'^3 X^4 \\
& + \frac{9}{4} \Sigma_k^5 X^5 - 17 \Sigma_k^7 \Sigma_k'^2 X^5 - \frac{5}{3} \Sigma_k^7 X^6 + \frac{20}{3} \Sigma_k^9 \Sigma_k'^2 X^6 \Big], \\
\Delta \tilde{K}_{43} = & \int \frac{d^4 k}{(2\pi)^4} \Big[-\frac{1}{12} \Sigma_k'^3 X + \frac{1}{48} \Sigma_k' X^2 + \frac{5}{8} \Sigma_k \Sigma_k'^2 X^2 + \frac{1}{3} \Sigma_k^2 \Sigma_k'^3 X^2 - \frac{1}{12} \Sigma_k X^3 - \frac{1}{12} \Sigma_k^2 \Sigma_k' X^3 \\
& - \frac{215}{24} \Sigma_k^3 \Sigma_k'^2 X^3 - \frac{5}{12} \Sigma_k^4 \Sigma_k'^3 X^3 - \frac{1}{4} \Sigma_k^3 X^4 + 29 \Sigma_k^5 \Sigma_k'^2 X^4 + \frac{1}{6} \Sigma_k^6 \Sigma_k'^3 X^4 + \frac{7}{2} \Sigma_k^5 X^5 \\
& - 34 \Sigma_k^7 \Sigma_k'^2 X^5 - \frac{10}{3} \Sigma_k^7 X^6 + \frac{40}{3} \Sigma_k^9 \Sigma_k'^2 X^6 \Big], \\
\Delta \tilde{K}_{44} = & \int \frac{d^4 k}{(2\pi)^4} \Big[\frac{1}{288} \Sigma_k''^2 + \frac{1}{960} \Sigma_k \Sigma_k''' X - \frac{1}{216} \Sigma_k'^4 X - \frac{5}{36} \Sigma_k^2 \Sigma_k''^2 X - \frac{13}{480} \Sigma_k'^2 X^2 + \frac{1}{240} \Sigma_k \Sigma_k'' X^2 \\
& + \frac{1}{160} \Sigma_k \Sigma_k'^3 X^2 - \frac{1}{960} \Sigma_k^3 \Sigma_k''' X^2 - \frac{217}{216} \Sigma_k^2 \Sigma_k'^4 X^2 + \frac{185}{288} \Sigma_k^4 \Sigma_k''^2 X^2 - \frac{1}{960} X^3 - \frac{1}{240} \Sigma_k \Sigma_k' X^3 \\
& + \frac{109}{64} \Sigma_k^2 \Sigma_k'^2 X^3 + \frac{1211}{480} \Sigma_k^3 \Sigma_k'^3 X^3 + \frac{1595}{216} \Sigma_k^4 \Sigma_k'^4 X^3 - \frac{163}{144} \Sigma_k^6 \Sigma_k''^2 X^3 + \frac{37}{960} \Sigma_k^2 X^4 - \frac{2147}{160} \Sigma_k^4 \Sigma_k'^2 X^4 \\
& - \frac{10501}{720} \Sigma_k^5 \Sigma_k'^3 X^4 - \frac{1445}{72} \Sigma_k^6 \Sigma_k'^4 X^4 + \frac{7}{8} \Sigma_k^8 \Sigma_k''^2 X^4 - \frac{43}{48} \Sigma_k^4 X^5 + \frac{407}{12} \Sigma_k^6 \Sigma_k'^2 X^5 + \frac{266}{9} \Sigma_k^7 \Sigma_k'^3 X^5 \\
& + \frac{937}{36} \Sigma_k^8 \Sigma_k'^4 X^5 - \frac{1}{4} \Sigma_k^{10} \Sigma_k''^2 X^5 + \frac{73}{24} \Sigma_k^6 X^6 - \frac{205}{6} \Sigma_k^8 \Sigma_k'^2 X^6 - \frac{51}{2} \Sigma_k^9 \Sigma_k'^3 X^6 - \frac{49}{3} \Sigma_k^{10} \Sigma_k'^4 X^6 \\
& - \frac{9}{4} \Sigma_k^8 X^7 + 12 \Sigma_k^{10} \Sigma_k'^2 X^7 + 8 \Sigma_k^{11} \Sigma_k'^3 X^7 + 4 \Sigma_k^{12} \Sigma_k'^4 X^7 \Big], \\
\Delta \tilde{K}_{45} = & \int \frac{d^4 k}{(2\pi)^4} \Big[-\frac{1}{288} \Sigma_k''^2 - \frac{1}{960} \Sigma_k \Sigma_k''' X + \frac{1}{216} \Sigma_k'^4 X + \frac{5}{36} \Sigma_k^2 \Sigma_k''^2 X - \frac{37}{480} \Sigma_k'^2 X^2 - \frac{1}{240} \Sigma_k \Sigma_k'' X^2 \\
& - \frac{1}{160} \Sigma_k \Sigma_k'^3 X^2 + \frac{1}{960} \Sigma_k^3 \Sigma_k''' X^2 + \frac{217}{216} \Sigma_k^2 \Sigma_k'^4 X^2 - \frac{185}{288} \Sigma_k^4 \Sigma_k''^2 X^2 + \frac{1}{960} X^3 + \frac{1}{240} \Sigma_k \Sigma_k' X^3 \\
& + \frac{353}{192} \Sigma_k^2 \Sigma_k'^2 X^3 - \frac{1211}{480} \Sigma_k^3 \Sigma_k'^3 X^3 - \frac{1595}{216} \Sigma_k^4 \Sigma_k'^4 X^3 + \frac{163}{144} \Sigma_k^6 \Sigma_k''^2 X^3 + \frac{1}{320} \Sigma_k^2 X^4 - \frac{1483}{160} \Sigma_k^4 \Sigma_k'^2 X^4 \\
& + \frac{10501}{720} \Sigma_k^5 \Sigma_k'^3 X^4 + \frac{1445}{72} \Sigma_k^6 \Sigma_k'^4 X^4 - \frac{7}{8} \Sigma_k^8 \Sigma_k''^2 X^4 - \frac{7}{16} \Sigma_k^4 X^5 + 19 \Sigma_k^6 \Sigma_k'^2 X^5 - \frac{266}{9} \Sigma_k^7 \Sigma_k'^3 X^5 \\
& - \frac{937}{36} \Sigma_k^8 \Sigma_k'^4 X^5 + \frac{1}{4} \Sigma_k^{10} \Sigma_k''^2 X^5 + \frac{19}{8} \Sigma_k^6 X^6 - \frac{35}{2} \Sigma_k^8 \Sigma_k'^2 X^6 + \frac{51}{2} \Sigma_k^9 \Sigma_k'^3 X^6 + \frac{49}{3} \Sigma_k^{10} \Sigma_k'^4 X^6
\end{aligned}$$

$$\begin{aligned}
& -\frac{9}{4}\Sigma_k^8 X^7 + 6\Sigma_k^{10}\Sigma_k'^2 X^7 - 8\Sigma_k^{11}\Sigma_k'^3 X^7 - 4\Sigma_k^{12}\Sigma_k'^4 X^7 \Big], \\
\Delta\tilde{K}_{46} = & \int \frac{d^4k}{(2\pi)^4} \Big[-\frac{1}{24}\Sigma_k''^2 - \frac{1}{960}\Sigma_k\Sigma_k''' X - \frac{13}{48}\Sigma_k'^4 X + \frac{11}{16}\Sigma_k^2\Sigma_k''^2 X - \frac{7}{480}\Sigma_k'^2 X^2 - \frac{1}{240}\Sigma_k\Sigma_k'' X^2 \\
& + \frac{277}{480}\Sigma_k\Sigma_k'^3 X^2 + \frac{1}{960}\Sigma_k^3\Sigma_k''' X^2 + \frac{281}{48}\Sigma_k^2\Sigma_k'^4 X^2 - \frac{45}{16}\Sigma_k^4\Sigma_k''^2 X^2 + \frac{41}{960}X^3 + \frac{41}{240}\Sigma_k\Sigma_k' X^3 \\
& + \frac{589}{192}\Sigma_k^2\Sigma_k'^2 X^3 - \frac{2117}{160}\Sigma_k^3\Sigma_k'^3 X^3 - \frac{1637}{48}\Sigma_k^4\Sigma_k'^4 X^3 + \frac{229}{48}\Sigma_k^6\Sigma_k''^2 X^3 - \frac{257}{960}\Sigma_k^2 X^4 - \frac{371}{20}\Sigma_k^4\Sigma_k'^2 X^4 \\
& + \frac{5159}{80}\Sigma_k^5\Sigma_k'^3 X^4 + \frac{12449}{144}\Sigma_k^6\Sigma_k'^4 X^4 - \frac{175}{48}\Sigma_k^8\Sigma_k''^2 X^4 - \frac{43}{48}\Sigma_k^4 X^5 + \frac{119}{3}\Sigma_k^6\Sigma_k'^2 X^5 - \frac{499}{4}\Sigma_k^7\Sigma_k'^3 X^5 \\
& - \frac{7871}{72}\Sigma_k^8\Sigma_k'^4 X^5 + \frac{25}{24}\Sigma_k^{10}\Sigma_k''^2 X^5 + \frac{145}{24}\Sigma_k^6 X^6 - \frac{110}{3}\Sigma_k^8\Sigma_k'^2 X^6 + \frac{425}{4}\Sigma_k^9\Sigma_k'^3 X^6 + \frac{1225}{18}\Sigma_k^{10}\Sigma_k'^4 X^6 \\
& - \frac{25}{4}\Sigma_k^8 X^7 + \frac{25}{2}\Sigma_k^{10}\Sigma_k'^2 X^7 - \frac{100}{3}\Sigma_k^{11}\Sigma_k'^3 X^7 - \frac{50}{3}\Sigma_k^{12}\Sigma_k'^4 X^7 \Big], \\
\Delta\tilde{K}_{47} = & \int \frac{d^4k}{(2\pi)^4} \Big[\frac{1}{48}\Sigma_k''^2 + \frac{67}{432}\Sigma_k'^4 X - \frac{41}{144}\Sigma_k^2\Sigma_k''^2 X + \frac{1}{24}\Sigma_k'^2 X^2 - \frac{11}{48}\Sigma_k\Sigma_k'^3 X^2 - \frac{1103}{432}\Sigma_k^2\Sigma_k'^4 X^2 \\
& + \frac{10}{9}\Sigma_k^4\Sigma_k''^2 X^2 - \frac{1}{48}X^3 - \frac{1}{12}\Sigma_k\Sigma_k' X^3 - \frac{223}{96}\Sigma_k^2\Sigma_k'^2 X^3 + \frac{187}{36}\Sigma_k^3\Sigma_k'^3 X^3 + \frac{5947}{432}\Sigma_k^4\Sigma_k'^4 X^3 \\
& - \frac{89}{48}\Sigma_k^6\Sigma_k''^2 X^3 + \frac{1}{6}\Sigma_k^2 X^4 + \frac{1297}{96}\Sigma_k^4\Sigma_k'^2 X^4 - \frac{3613}{144}\Sigma_k^5\Sigma_k'^3 X^4 - \frac{14653}{432}\Sigma_k^6\Sigma_k'^4 X^4 + \frac{203}{144}\Sigma_k^8\Sigma_k''^2 X^4 \\
& + \frac{13}{16}\Sigma_k^4 X^5 - \frac{1367}{48}\Sigma_k^6\Sigma_k'^2 X^5 + \frac{3479}{72}\Sigma_k^7\Sigma_k'^3 X^5 + \frac{9163}{216}\Sigma_k^8\Sigma_k'^4 X^5 - \frac{29}{72}\Sigma_k^{10}\Sigma_k''^2 X^5 - \frac{59}{16}\Sigma_k^6 X^6 \\
& + \frac{155}{6}\Sigma_k^8\Sigma_k'^2 X^6 - \frac{493}{12}\Sigma_k^9\Sigma_k'^3 X^6 - \frac{1421}{54}\Sigma_k^{10}\Sigma_k'^4 X^6 + \frac{161}{48}\Sigma_k^8 X^7 - \frac{103}{12}\Sigma_k^{10}\Sigma_k'^2 X^7 + \frac{116}{9}\Sigma_k^{11}\Sigma_k'^3 X^7 \\
& + \frac{58}{9}\Sigma_k^{12}\Sigma_k'^4 X^7 \Big], \\
\Delta\tilde{K}_{48} = & \int \frac{d^4k}{(2\pi)^4} \Big[\frac{1}{24}\Sigma_k''^2 + \frac{23}{72}\Sigma_k'^4 X - \frac{13}{24}\Sigma_k^2\Sigma_k''^2 X - \frac{1}{3}\Sigma_k\Sigma_k'^3 X^2 - \frac{355}{72}\Sigma_k^2\Sigma_k'^4 X^2 + \frac{25}{12}\Sigma_k^4\Sigma_k''^2 X^2 \\
& - \frac{1}{24}X^3 - \frac{1}{6}\Sigma_k\Sigma_k' X^3 - \frac{9}{2}\Sigma_k^2\Sigma_k'^2 X^3 + \frac{28}{3}\Sigma_k^3\Sigma_k'^3 X^3 + \frac{1871}{72}\Sigma_k^4\Sigma_k'^4 X^3 - \frac{83}{24}\Sigma_k^6\Sigma_k''^2 X^3 \\
& + \frac{3}{8}\Sigma_k^2 X^4 + 33\Sigma_k^4\Sigma_k'^2 X^4 - \frac{139}{3}\Sigma_k^5\Sigma_k'^3 X^4 - \frac{1523}{24}\Sigma_k^6\Sigma_k'^4 X^4 + \frac{21}{8}\Sigma_k^8\Sigma_k''^2 X^4 + \frac{13}{8}\Sigma_k^4 X^5 \\
& - 80\Sigma_k^6\Sigma_k'^2 X^5 + \frac{539}{6}\Sigma_k^7\Sigma_k'^3 X^5 + \frac{949}{12}\Sigma_k^8\Sigma_k'^4 X^5 - \frac{3}{4}\Sigma_k^{10}\Sigma_k''^2 X^5 - \frac{77}{8}\Sigma_k^6 X^6 + 80\Sigma_k^8\Sigma_k'^2 X^6 \\
& - \frac{153}{2}\Sigma_k^9\Sigma_k'^3 X^6 - 49\Sigma_k^{10}\Sigma_k'^4 X^6 + \frac{75}{8}\Sigma_k^8 X^7 - \frac{57}{2}\Sigma_k^{10}\Sigma_k'^2 X^7 + 24\Sigma_k^{11}\Sigma_k'^3 X^7 + 12\Sigma_k^{12}\Sigma_k'^4 X^7 \Big], \\
\Delta\tilde{K}_{49} = & \int \frac{d^4k}{(2\pi)^4} \Big[-\frac{1}{36}\Sigma_k''^2 - \frac{1}{480}\Sigma_k\Sigma_k''' X - \frac{31}{216}\Sigma_k'^4 X + \frac{41}{72}\Sigma_k^2\Sigma_k''^2 X + \frac{13}{240}\Sigma_k'^2 X^2 - \frac{1}{120}\Sigma_k\Sigma_k'' X^2 \\
& + \frac{77}{240}\Sigma_k\Sigma_k'^3 X^2 + \frac{1}{480}\Sigma_k^3\Sigma_k''' X^2 + \frac{995}{216}\Sigma_k^2\Sigma_k'^4 X^2 - \frac{175}{72}\Sigma_k^4\Sigma_k''^2 X^2 + \frac{1}{480}X^3 + \frac{1}{120}\Sigma_k\Sigma_k' X^3 \\
& + \frac{51}{32}\Sigma_k^2\Sigma_k'^2 X^3 - \frac{2591}{240}\Sigma_k^3\Sigma_k'^3 X^3 - \frac{6247}{216}\Sigma_k^4\Sigma_k'^4 X^3 + \frac{301}{72}\Sigma_k^6\Sigma_k''^2 X^3 - \frac{217}{480}\Sigma_k^2 X^4 - \frac{1173}{80}\Sigma_k^4\Sigma_k'^2 X^4 \\
& + \frac{20011}{360}\Sigma_k^5\Sigma_k'^3 X^4 + \frac{5411}{72}\Sigma_k^6\Sigma_k'^4 X^4 - \frac{77}{24}\Sigma_k^8\Sigma_k''^2 X^4 + \frac{1}{6}\Sigma_k^4 X^5 + \frac{235}{6}\Sigma_k^6\Sigma_k'^2 X^5 - \frac{1967}{18}\Sigma_k^7\Sigma_k'^3 X^5 \\
& - \frac{1151}{12}\Sigma_k^8\Sigma_k'^4 X^5 + \frac{11}{12}\Sigma_k^{10}\Sigma_k''^2 X^5 + \frac{139}{24}\Sigma_k^6 X^6 - \frac{125}{3}\Sigma_k^8\Sigma_k'^2 X^6 + \frac{187}{2}\Sigma_k^9\Sigma_k'^3 X^6 + \frac{539}{9}\Sigma_k^{10}\Sigma_k'^4 X^6 \\
& - \frac{53}{8}\Sigma_k^8 X^7 + \frac{31}{2}\Sigma_k^{10}\Sigma_k'^2 X^7 - \frac{88}{3}\Sigma_k^{11}\Sigma_k'^3 X^7 - \frac{44}{3}\Sigma_k^{12}\Sigma_k'^4 X^7 \Big], \\
\Delta\tilde{K}_{50} = & \int \frac{d^4k}{(2\pi)^4} \Big[-\frac{1}{36}\Sigma_k''^2 - \frac{11}{54}\Sigma_k'^4 X + \frac{7}{18}\Sigma_k^2\Sigma_k''^2 X - \frac{1}{32}\Sigma_k'^2 X^2 + \frac{17}{48}\Sigma_k\Sigma_k'^3 X^2 + \frac{187}{54}\Sigma_k^2\Sigma_k'^4 X^2 \\
& - \frac{55}{36}\Sigma_k^4\Sigma_k''^2 X^2 + \frac{1}{24}X^3 + \frac{1}{6}\Sigma_k\Sigma_k' X^3 + \frac{331}{96}\Sigma_k^2\Sigma_k'^2 X^3 - \frac{263}{36}\Sigma_k^3\Sigma_k'^3 X^3 - \frac{1019}{54}\Sigma_k^4\Sigma_k'^4 X^3
\end{aligned}$$

$$\begin{aligned}
& + \frac{23}{9} \Sigma_k^6 \Sigma_k^{\prime\prime 2} X^3 - \frac{3}{16} \Sigma_k^2 X^4 - \frac{2129}{96} \Sigma_k^4 \Sigma_k^{\prime 2} X^4 + \frac{1669}{48} \Sigma_k^5 \Sigma_k^{\prime 3} X^4 + \frac{2521}{54} \Sigma_k^6 \Sigma_k^{\prime 4} X^4 - \frac{35}{18} \Sigma_k^8 \Sigma_k^{\prime 2} X^4 \\
& - \frac{65}{48} \Sigma_k^4 X^5 + \frac{2393}{48} \Sigma_k^6 \Sigma_k^{\prime 2} X^5 - \frac{1601}{24} \Sigma_k^7 \Sigma_k^{\prime 3} X^5 - \frac{1579}{27} \Sigma_k^8 \Sigma_k^{\prime 4} X^5 + \frac{5}{9} \Sigma_k^{10} \Sigma_k^{\prime 2} X^5 + \frac{295}{48} \Sigma_k^6 X^6 \\
& - \frac{95}{2} \Sigma_k^8 \Sigma_k^{\prime 2} X^6 + \frac{170}{3} \Sigma_k^9 \Sigma_k^{\prime 3} X^6 + \frac{980}{27} \Sigma_k^{10} \Sigma_k^{\prime 4} X^6 - \frac{277}{48} \Sigma_k^8 X^7 + \frac{197}{12} \Sigma_k^{10} \Sigma_k^{\prime 2} X^7 - \frac{160}{9} \Sigma_k^{11} \Sigma_k^{\prime 3} X^7 \\
& - \frac{80}{9} \Sigma_k^{12} \Sigma_k^{\prime 4} X^7 \Big], \\
\Delta \tilde{K}_{51} = & \int \frac{d^4 k}{(2\pi)^4} \Big[\frac{11}{288} \Sigma_k^{\prime\prime 2} + \frac{1}{480} \Sigma_k \Sigma_k^{\prime\prime\prime} X + \frac{2}{9} \Sigma_k^{\prime 4} X - \frac{17}{24} \Sigma_k^2 \Sigma_k^{\prime\prime 2} X - \frac{43}{240} \Sigma_k^{\prime 2} X^2 + \frac{1}{120} \Sigma_k \Sigma_k^{\prime\prime} X^2 \\
& - \frac{137}{240} \Sigma_k \Sigma_k^{\prime 3} X^2 - \frac{1}{480} \Sigma_k^3 \Sigma_k^{\prime\prime\prime} X^2 - \frac{211}{36} \Sigma_k^2 \Sigma_k^{\prime 4} X^2 + \frac{95}{32} \Sigma_k^4 \Sigma_k^{\prime\prime 2} X^2 - \frac{11}{480} X^3 - \frac{11}{120} \Sigma_k \Sigma_k^{\prime} X^3 \\
& + \frac{379}{96} \Sigma_k^2 \Sigma_k^{\prime 2} X^3 + \frac{9983}{720} \Sigma_k^3 \Sigma_k^{\prime 3} X^3 + \frac{641}{18} \Sigma_k^4 \Sigma_k^{\prime 4} X^3 - \frac{731}{144} \Sigma_k^6 \Sigma_k^{\prime\prime 2} X^3 + \frac{17}{480} \Sigma_k^2 X^4 - \frac{6641}{240} \Sigma_k^4 \Sigma_k^{\prime 2} X^4 \\
& - \frac{12313}{180} \Sigma_k^5 \Sigma_k^{\prime 3} X^4 - \frac{9889}{108} \Sigma_k^6 \Sigma_k^{\prime 4} X^4 + \frac{35}{9} \Sigma_k^8 \Sigma_k^{\prime\prime 2} X^4 - \frac{3}{2} \Sigma_k^4 X^5 + \frac{847}{12} \Sigma_k^6 \Sigma_k^{\prime 2} X^5 + \frac{1196}{9} \Sigma_k^7 \Sigma_k^{\prime 3} X^5 \\
& + \frac{3143}{27} \Sigma_k^8 \Sigma_k^{\prime 4} X^5 - \frac{10}{9} \Sigma_k^{10} \Sigma_k^{\prime\prime 2} X^5 + 5 \Sigma_k^6 X^6 - \frac{220}{3} \Sigma_k^8 \Sigma_k^{\prime 2} X^6 - \frac{340}{3} \Sigma_k^9 \Sigma_k^{\prime 3} X^6 - \frac{1960}{27} \Sigma_k^{10} \Sigma_k^{\prime 4} X^6 \\
& - \frac{10}{3} \Sigma_k^8 X^7 + \frac{80}{3} \Sigma_k^{10} \Sigma_k^{\prime 2} X^7 + \frac{320}{9} \Sigma_k^{11} \Sigma_k^{\prime 3} X^7 + \frac{160}{9} \Sigma_k^{12} \Sigma_k^{\prime 4} X^7 \Big], \\
\Delta \tilde{K}_{52} = & \int \frac{d^4 k}{(2\pi)^4} \Big[-\frac{5}{288} \Sigma_k^{\prime\prime 2} - \frac{1}{480} \Sigma_k \Sigma_k^{\prime\prime\prime} X - \frac{5}{72} \Sigma_k^{\prime 4} X + \frac{5}{12} \Sigma_k^2 \Sigma_k^{\prime\prime 2} X - \frac{7}{240} \Sigma_k^{\prime 2} X^2 - \frac{1}{120} \Sigma_k \Sigma_k^{\prime\prime} X^2 \\
& + \frac{19}{80} \Sigma_k \Sigma_k^{\prime 3} X^2 + \frac{1}{480} \Sigma_k^3 \Sigma_k^{\prime\prime\prime} X^2 + \frac{235}{72} \Sigma_k^2 \Sigma_k^{\prime 4} X^2 - \frac{175}{96} \Sigma_k^4 \Sigma_k^{\prime\prime 2} X^2 + \frac{11}{480} X^3 + \frac{11}{120} \Sigma_k \Sigma_k^{\prime} X^3 \\
& + \frac{7}{32} \Sigma_k^2 \Sigma_k^{\prime 2} X^3 - \frac{5843}{720} \Sigma_k^3 \Sigma_k^{\prime 3} X^3 - \frac{515}{24} \Sigma_k^4 \Sigma_k^{\prime 4} X^3 + \frac{455}{144} \Sigma_k^6 \Sigma_k^{\prime\prime 2} X^3 + \frac{1}{160} \Sigma_k^2 X^4 + \frac{1931}{240} \Sigma_k^4 \Sigma_k^{\prime 2} X^4 \\
& + \frac{3779}{90} \Sigma_k^5 \Sigma_k^{\prime 3} X^4 + \frac{12215}{216} \Sigma_k^6 \Sigma_k^{\prime 4} X^4 - \frac{175}{72} \Sigma_k^8 \Sigma_k^{\prime\prime 2} X^4 - \frac{1}{12} \Sigma_k^4 X^5 - \frac{133}{4} \Sigma_k^6 \Sigma_k^{\prime 2} X^5 - \frac{1489}{18} \Sigma_k^7 \Sigma_k^{\prime 3} X^5 \\
& - \frac{7835}{108} \Sigma_k^8 \Sigma_k^{\prime 4} X^5 + \frac{25}{36} \Sigma_k^{10} \Sigma_k^{\prime\prime 2} X^5 - \frac{25}{12} \Sigma_k^6 X^6 + \frac{125}{3} \Sigma_k^8 \Sigma_k^{\prime 2} X^6 + \frac{425}{6} \Sigma_k^9 \Sigma_k^{\prime 3} X^6 + \frac{1225}{27} \Sigma_k^{10} \Sigma_k^{\prime 4} X^6 \\
& + \frac{25}{12} \Sigma_k^8 X^7 - \frac{50}{3} \Sigma_k^{10} \Sigma_k^{\prime 2} X^7 - \frac{200}{9} \Sigma_k^{11} \Sigma_k^{\prime 3} X^7 - \frac{100}{9} \Sigma_k^{12} \Sigma_k^{\prime 4} X^7 \Big], \\
\Delta \tilde{K}_{53} = & \int \frac{d^4 k}{(2\pi)^4} \Big[\frac{33}{32} \Sigma_k \Sigma_k^{\prime 2} X^2 - \frac{7}{96} \Sigma_k X^3 - \frac{977}{96} \Sigma_k^3 \Sigma_k^{\prime 2} X^3 + \frac{1}{96} \Sigma_k^3 X^4 + \frac{1375}{48} \Sigma_k^5 \Sigma_k^{\prime 2} X^4 + \frac{61}{24} \Sigma_k^5 X^5 \\
& - \frac{187}{6} \Sigma_k^7 \Sigma_k^{\prime 2} X^5 - \frac{35}{12} \Sigma_k^7 X^6 + \frac{35}{3} \Sigma_k^9 \Sigma_k^{\prime 2} X^6 \Big], \\
\Delta \tilde{K}_{54} = & \int \frac{d^4 k}{(2\pi)^4} \Big[\frac{1}{48} \Sigma_k^{\prime 3} X + \frac{1}{96} \Sigma_k^{\prime} X^2 + \frac{1}{8} \Sigma_k \Sigma_k^{\prime 2} X^2 - \frac{1}{12} \Sigma_k^2 \Sigma_k^{\prime 3} X^2 - \frac{1}{24} \Sigma_k^2 \Sigma_k^{\prime} X^3 - \frac{43}{48} \Sigma_k^3 \Sigma_k^{\prime 2} X^3 \\
& + \frac{5}{48} \Sigma_k^4 \Sigma_k^{\prime 3} X^3 - \frac{1}{24} \Sigma_k^3 X^4 + \frac{77}{48} \Sigma_k^5 \Sigma_k^{\prime 2} X^4 - \frac{1}{24} \Sigma_k^6 \Sigma_k^{\prime 3} X^4 + \frac{5}{24} \Sigma_k^5 X^5 - \frac{5}{6} \Sigma_k^7 \Sigma_k^{\prime 2} X^5 \Big], \\
\Delta \tilde{K}_{55} = & \int \frac{d^4 k}{(2\pi)^4} \Big[-\frac{7}{576} \Sigma_k^{\prime\prime 2} + \frac{1}{960} \Sigma_k \Sigma_k^{\prime\prime\prime} X - \frac{17}{144} \Sigma_k^{\prime 4} X + \frac{1}{12} \Sigma_k^2 \Sigma_k^{\prime\prime 2} X - \frac{11}{160} \Sigma_k^{\prime 2} X^2 + \frac{1}{240} \Sigma_k \Sigma_k^{\prime\prime} X^2 \\
& + \frac{143}{480} \Sigma_k \Sigma_k^{\prime 3} X^2 - \frac{1}{960} \Sigma_k^3 \Sigma_k^{\prime\prime\prime} X^2 + \frac{139}{144} \Sigma_k^2 \Sigma_k^{\prime 4} X^2 - \frac{15}{64} \Sigma_k^4 \Sigma_k^{\prime\prime 2} X^2 - \frac{11}{960} X^3 - \frac{11}{240} \Sigma_k \Sigma_k^{\prime} X^3 \\
& + \frac{83}{192} \Sigma_k^2 \Sigma_k^{\prime 2} X^3 - \frac{2917}{1440} \Sigma_k^3 \Sigma_k^{\prime 3} X^3 - \frac{493}{144} \Sigma_k^4 \Sigma_k^{\prime 4} X^3 + \frac{97}{288} \Sigma_k^6 \Sigma_k^{\prime\prime 2} X^3 + \frac{37}{960} \Sigma_k^2 X^4 - \frac{431}{480} \Sigma_k^4 \Sigma_k^{\prime 2} X^4 \\
& + \frac{1051}{180} \Sigma_k^5 \Sigma_k^{\prime 3} X^4 + \frac{2911}{432} \Sigma_k^6 \Sigma_k^{\prime 4} X^4 - \frac{35}{144} \Sigma_k^8 \Sigma_k^{\prime\prime 2} X^4 - \frac{1}{6} \Sigma_k^4 X^5 - \frac{7}{24} \Sigma_k^6 \Sigma_k^{\prime 2} X^5 - \frac{323}{36} \Sigma_k^7 \Sigma_k^{\prime 3} X^5 \\
& - \frac{1639}{216} \Sigma_k^8 \Sigma_k^{\prime 4} X^5 + \frac{5}{72} \Sigma_k^{10} \Sigma_k^{\prime\prime 2} X^5 + \frac{5}{24} \Sigma_k^6 X^6 + \frac{5}{2} \Sigma_k^8 \Sigma_k^{\prime 2} X^6 + \frac{85}{12} \Sigma_k^9 \Sigma_k^{\prime 3} X^6 + \frac{245}{54} \Sigma_k^{10} \Sigma_k^{\prime 4} X^6 \\
& + \frac{5}{24} \Sigma_k^8 X^7 - \frac{5}{3} \Sigma_k^{10} \Sigma_k^{\prime 2} X^7 - \frac{20}{9} \Sigma_k^{11} \Sigma_k^{\prime 3} X^7 - \frac{10}{9} \Sigma_k^{12} \Sigma_k^{\prime 4} X^7 \Big],
\end{aligned}$$

$$\begin{aligned}
\Delta \tilde{K}_{56} &= \int \frac{d^4 k}{(2\pi)^4} \left[\frac{1}{12} \Sigma_k'^3 X - \frac{1}{192} \Sigma_k' X^2 - \frac{1}{4} \Sigma_k \Sigma_k'^2 X^2 - \frac{1}{3} \Sigma_k^2 \Sigma_k'^3 X^2 + \frac{1}{192} \Sigma_k X^3 + \frac{1}{48} \Sigma_k^2 \Sigma_k' X^3 \right. \\
&\quad + \frac{113}{24} \Sigma_k^3 \Sigma_k'^2 X^3 + \frac{5}{12} \Sigma_k^4 \Sigma_k'^3 X^3 + \frac{27}{64} \Sigma_k^3 X^4 - \frac{133}{8} \Sigma_k^5 \Sigma_k'^2 X^4 - \frac{1}{6} \Sigma_k^6 \Sigma_k'^3 X^4 - \frac{19}{8} \Sigma_k^5 X^5 \\
&\quad \left. + \frac{41}{2} \Sigma_k^7 \Sigma_k'^2 X^5 + \frac{25}{12} \Sigma_k^7 X^6 - \frac{25}{3} \Sigma_k^9 \Sigma_k'^2 X^6 \right], \\
\Delta \tilde{K}_{57} &= \int \frac{d^4 k}{(2\pi)^4} \left[\frac{7}{192} \Sigma_k''^2 + \frac{1}{640} \Sigma_k \Sigma_k''' X + \frac{11}{48} \Sigma_k'^4 X - \frac{5}{8} \Sigma_k^2 \Sigma_k''^2 X - \frac{3}{320} \Sigma_k'^2 X^2 + \frac{1}{160} \Sigma_k \Sigma_k'' X^2 \right. \\
&\quad - \frac{157}{320} \Sigma_k \Sigma_k'^3 X^2 - \frac{1}{640} \Sigma_k^3 \Sigma_k''' X^2 - \frac{253}{48} \Sigma_k^2 \Sigma_k'^4 X^2 + \frac{165}{64} \Sigma_k^4 \Sigma_k''^2 X^2 - \frac{23}{1920} X^3 - \frac{23}{480} \Sigma_k \Sigma_k' X^3 \\
&\quad + \frac{341}{384} \Sigma_k^2 \Sigma_k'^2 X^3 + \frac{11513}{960} \Sigma_k^3 \Sigma_k'^3 X^3 + \frac{1495}{48} \Sigma_k^4 \Sigma_k'^4 X^3 - \frac{421}{96} \Sigma_k^6 \Sigma_k''^2 X^3 + \frac{151}{1920} \Sigma_k^2 X^4 - \frac{3491}{320} \Sigma_k^4 \Sigma_k'^2 X^4 \\
&\quad - \frac{28361}{480} \Sigma_k^5 \Sigma_k'^3 X^4 - \frac{11425}{144} \Sigma_k^6 \Sigma_k'^4 X^4 + \frac{161}{48} \Sigma_k^8 \Sigma_k''^2 X^4 - \frac{67}{96} \Sigma_k^4 X^5 + \frac{265}{8} \Sigma_k^6 \Sigma_k'^2 X^5 + \frac{344}{3} \Sigma_k^7 \Sigma_k'^3 X^5 \\
&\quad + \frac{7237}{72} \Sigma_k^8 \Sigma_k'^4 X^5 - \frac{23}{24} \Sigma_k^{10} \Sigma_k''^2 X^5 + \frac{73}{48} \Sigma_k^6 X^6 - \frac{445}{12} \Sigma_k^8 \Sigma_k'^2 X^6 - \frac{391}{4} \Sigma_k^9 \Sigma_k'^3 X^6 - \frac{1127}{18} \Sigma_k^{10} \Sigma_k'^4 X^6 \\
&\quad \left. - \frac{5}{8} \Sigma_k^8 X^7 + 14 \Sigma_k^{10} \Sigma_k'^2 X^7 + \frac{92}{3} \Sigma_k^{11} \Sigma_k'^3 X^7 + \frac{46}{3} \Sigma_k^{12} \Sigma_k'^4 X^7 \right], \\
\Delta \tilde{K}_{58} &= \int \frac{d^4 k}{(2\pi)^4} \left[-\frac{3}{2} \Sigma_k \Sigma_k'^2 X^2 + \frac{5}{48} \Sigma_k X^3 + \frac{353}{24} \Sigma_k^3 \Sigma_k'^2 X^3 + \frac{1}{48} \Sigma_k^3 X^4 - \frac{989}{24} \Sigma_k^5 \Sigma_k'^2 X^4 - \frac{11}{3} \Sigma_k^5 X^5 \right. \\
&\quad \left. + \frac{134}{3} \Sigma_k^7 \Sigma_k'^2 X^5 + \frac{25}{6} \Sigma_k^7 X^6 - \frac{50}{3} \Sigma_k^9 \Sigma_k'^2 X^6 \right], \\
\Delta \tilde{K}_{59} &= \int \frac{d^4 k}{(2\pi)^4} \left[-\frac{1}{48} \Sigma_k''^2 - \frac{1}{8} \Sigma_k'^4 X + \frac{3}{8} \Sigma_k^2 \Sigma_k''^2 X + \frac{1}{2} \Sigma_k \Sigma_k'^3 X^2 + \frac{25}{8} \Sigma_k^2 \Sigma_k'^4 X^2 - \frac{25}{16} \Sigma_k^4 \Sigma_k''^2 X^2 \right. \\
&\quad - \frac{3}{4} \Sigma_k^2 \Sigma_k'^2 X^3 - \frac{97}{12} \Sigma_k^3 \Sigma_k'^3 X^3 - \frac{451}{24} \Sigma_k^4 \Sigma_k'^4 X^3 + \frac{8}{3} \Sigma_k^6 \Sigma_k''^2 X^3 + \frac{37}{4} \Sigma_k^4 \Sigma_k'^2 X^4 + \frac{443}{12} \Sigma_k^5 \Sigma_k'^3 X^4 \\
&\quad + \frac{3467}{72} \Sigma_k^6 \Sigma_k'^4 X^4 - \frac{49}{24} \Sigma_k^8 \Sigma_k''^2 X^4 + \frac{1}{4} \Sigma_k^4 X^5 - \frac{59}{2} \Sigma_k^6 \Sigma_k'^2 X^5 - \frac{421}{6} \Sigma_k^7 \Sigma_k'^3 X^5 - \frac{2201}{36} \Sigma_k^8 \Sigma_k'^4 X^5 \\
&\quad + \frac{7}{12} \Sigma_k^{10} \Sigma_k''^2 X^5 - \frac{7}{4} \Sigma_k^6 X^6 + 35 \Sigma_k^8 \Sigma_k'^2 X^6 + \frac{119}{2} \Sigma_k^9 \Sigma_k'^3 X^6 + \frac{343}{9} \Sigma_k^{10} \Sigma_k'^4 X^6 + \frac{7}{4} \Sigma_k^8 X^7 \\
&\quad \left. - 14 \Sigma_k^{10} \Sigma_k'^2 X^7 - \frac{56}{3} \Sigma_k^{11} \Sigma_k'^3 X^7 - \frac{28}{3} \Sigma_k^{12} \Sigma_k'^4 X^7 \right], \\
\Delta \tilde{K}_{60} &= \int \frac{d^4 k}{(2\pi)^4} \left[\frac{1}{144} \Sigma_k''^2 - \frac{1}{240} \Sigma_k \Sigma_k''' X + \frac{7}{54} \Sigma_k'^4 X + \frac{5}{36} \Sigma_k^2 \Sigma_k''^2 X + \frac{1}{40} \Sigma_k'^2 X^2 - \frac{1}{60} \Sigma_k \Sigma_k'' X^2 \right. \\
&\quad - \frac{83}{120} \Sigma_k \Sigma_k'^3 X^2 + \frac{1}{240} \Sigma_k^3 \Sigma_k''' X^2 + \frac{17}{27} \Sigma_k^2 \Sigma_k'^4 X^2 - \frac{115}{144} \Sigma_k^4 \Sigma_k''^2 X^2 + \frac{11}{240} X^3 + \frac{11}{60} \Sigma_k \Sigma_k' X^3 \\
&\quad + \frac{1}{48} \Sigma_k^2 \Sigma_k'^2 X^3 - \frac{61}{120} \Sigma_k^3 \Sigma_k'^3 X^3 - \frac{455}{54} \Sigma_k^4 \Sigma_k'^4 X^3 + \frac{107}{72} \Sigma_k^6 \Sigma_k''^2 X^3 - \frac{17}{240} \Sigma_k^2 X^4 + \frac{237}{40} \Sigma_k^4 \Sigma_k'^2 X^4 \\
&\quad + \frac{709}{45} \Sigma_k^5 \Sigma_k'^3 X^4 + \frac{230}{9} \Sigma_k^6 \Sigma_k'^4 X^4 - \frac{7}{6} \Sigma_k^8 \Sigma_k''^2 X^4 + \frac{1}{6} \Sigma_k^4 X^5 - \frac{64}{3} \Sigma_k^6 \Sigma_k'^2 X^5 - \frac{341}{9} \Sigma_k^7 \Sigma_k'^3 X^5 \\
&\quad - \frac{103}{3} \Sigma_k^8 \Sigma_k'^4 X^5 + \frac{1}{3} \Sigma_k^{10} \Sigma_k''^2 X^5 - \frac{11}{6} \Sigma_k^6 X^6 + \frac{70}{3} \Sigma_k^8 \Sigma_k'^2 X^6 + 34 \Sigma_k^9 \Sigma_k'^3 X^6 + \frac{196}{9} \Sigma_k^{10} \Sigma_k'^4 X^6 \\
&\quad \left. + \Sigma_k^8 X^7 - 8 \Sigma_k^{10} \Sigma_k'^2 X^7 - \frac{32}{3} \Sigma_k^{11} \Sigma_k'^3 X^7 - \frac{16}{3} \Sigma_k^{12} \Sigma_k'^4 X^7 \right], \\
\Delta \tilde{K}_{61} &= \int \frac{d^4 k}{(2\pi)^4} \left[\frac{1}{48} \Sigma_k''^2 + \frac{1}{240} \Sigma_k \Sigma_k''' X + \frac{7}{216} \Sigma_k'^4 X - \frac{47}{72} \Sigma_k^2 \Sigma_k''^2 X - \frac{1}{15} \Sigma_k'^2 X^2 + \frac{1}{60} \Sigma_k \Sigma_k'' X^2 \right. \\
&\quad - \frac{37}{120} \Sigma_k \Sigma_k'^3 X^2 - \frac{1}{240} \Sigma_k^3 \Sigma_k''' X^2 - \frac{1055}{216} \Sigma_k^2 \Sigma_k'^4 X^2 + \frac{425}{144} \Sigma_k^4 \Sigma_k''^2 X^2 + \frac{3}{80} X^3 + \frac{3}{20} \Sigma_k \Sigma_k' X^3 \\
&\quad + \frac{47}{16} \Sigma_k^2 \Sigma_k'^2 X^3 + \frac{4643}{360} \Sigma_k^3 \Sigma_k'^3 X^3 + \frac{7399}{216} \Sigma_k^4 \Sigma_k'^4 X^3 - \frac{31}{6} \Sigma_k^6 \Sigma_k''^2 X^3 - \frac{21}{80} \Sigma_k^2 X^4 - \frac{6037}{240} \Sigma_k^4 \Sigma_k'^2 X^4 \\
&\quad \left. - \frac{6143}{90} \Sigma_k^5 \Sigma_k'^3 X^4 - \frac{19861}{216} \Sigma_k^6 \Sigma_k'^4 X^4 + \frac{287}{72} \Sigma_k^8 \Sigma_k''^2 X^4 - \frac{4}{3} \Sigma_k^4 X^5 + \frac{205}{3} \Sigma_k^6 \Sigma_k'^2 X^5 + \frac{2437}{18} \Sigma_k^7 \Sigma_k'^3 X^5 \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{12823}{108} \Sigma_k^8 \Sigma_k'^4 X^5 - \frac{41}{36} \Sigma_k^{10} \Sigma_k''^2 X^5 + \frac{14}{3} \Sigma_k^6 X^6 - \frac{220}{3} \Sigma_k^8 \Sigma_k'^2 X^6 - \frac{697}{6} \Sigma_k^9 \Sigma_k'^3 X^6 - \frac{2009}{27} \Sigma_k^{10} \Sigma_k'^4 X^6 \\
& - \frac{41}{12} \Sigma_k^8 X^7 + \frac{82}{3} \Sigma_k^{10} \Sigma_k'^2 X^7 + \frac{328}{9} \Sigma_k^{11} \Sigma_k'^3 X^7 + \frac{164}{9} \Sigma_k^{12} \Sigma_k'^4 X^7 \Big], \\
\Delta \tilde{K}_{62} = & \int \frac{d^4 k}{(2\pi)^4} \Big[-\frac{1}{36} \Sigma_k''^2 - \frac{47}{216} \Sigma_k'^4 X + \frac{25}{72} \Sigma_k^2 \Sigma_k''^2 X + \frac{1}{8} \Sigma_k'^2 X^2 + \frac{1}{3} \Sigma_k \Sigma_k'^3 X^2 + \frac{691}{216} \Sigma_k^2 \Sigma_k'^4 X^2 \\
& - \frac{95}{72} \Sigma_k^4 \Sigma_k''^2 X^2 - \frac{47}{24} \Sigma_k^2 \Sigma_k'^2 X^3 - \frac{115}{18} \Sigma_k^3 \Sigma_k'^3 X^3 - \frac{3575}{216} \Sigma_k^4 \Sigma_k'^4 X^3 + \frac{157}{72} \Sigma_k^6 \Sigma_k''^2 X^3 - \frac{1}{6} \Sigma_k^2 X^4 \\
& + \frac{551}{48} \Sigma_k^4 \Sigma_k'^2 X^4 + \frac{179}{6} \Sigma_k^5 \Sigma_k'^3 X^4 + \frac{8665}{216} \Sigma_k^6 \Sigma_k'^4 X^4 - \frac{119}{72} \Sigma_k^8 \Sigma_k''^2 X^4 + \frac{2}{3} \Sigma_k^4 X^5 - \frac{85}{3} \Sigma_k^6 \Sigma_k'^2 X^5 \\
& - \frac{341}{6} \Sigma_k^7 \Sigma_k'^3 X^5 - \frac{5383}{108} \Sigma_k^8 \Sigma_k'^4 X^5 + \frac{17}{36} \Sigma_k^{10} \Sigma_k''^2 X^5 - \frac{11}{6} \Sigma_k^6 X^6 + 30 \Sigma_k^8 \Sigma_k'^2 X^6 + \frac{289}{6} \Sigma_k^9 \Sigma_k'^3 X^6 \\
& + \frac{833}{27} \Sigma_k^{10} \Sigma_k'^4 X^6 + \frac{17}{12} \Sigma_k^8 X^7 - \frac{34}{3} \Sigma_k^{10} \Sigma_k'^2 X^7 - \frac{136}{9} \Sigma_k^{11} \Sigma_k'^3 X^7 - \frac{68}{9} \Sigma_k^{12} \Sigma_k'^4 X^7 \Big], \\
\Delta \tilde{K}_{63} = & \int \frac{d^4 k}{(2\pi)^4} \Big[-\frac{1}{8} \Sigma_k'^3 X + \frac{1}{2} \Sigma_k^2 \Sigma_k'^3 X^2 + \frac{5}{8} \Sigma_k^3 \Sigma_k'^2 X^3 - \frac{5}{8} \Sigma_k^4 \Sigma_k'^3 X^3 - \frac{13}{8} \Sigma_k^5 \Sigma_k'^2 X^4 + \frac{1}{4} \Sigma_k^6 \Sigma_k'^3 X^4 \\
& - \frac{1}{4} \Sigma_k^5 X^5 + \Sigma_k^7 \Sigma_k'^2 X^5 \Big], \\
\Delta \tilde{K}_{64} = & \int \frac{d^4 k}{(2\pi)^4} \Big[-\frac{1}{96} \Sigma_k''^2 - \frac{13}{144} \Sigma_k'^4 X + \frac{5}{48} \Sigma_k^2 \Sigma_k''^2 X + \frac{1}{4} \Sigma_k \Sigma_k'^3 X^2 + \frac{149}{144} \Sigma_k^2 \Sigma_k'^4 X^2 - \frac{35}{96} \Sigma_k^4 \Sigma_k''^2 X^2 \\
& - \frac{3}{8} \Sigma_k^2 \Sigma_k'^2 X^3 - \frac{19}{8} \Sigma_k^3 \Sigma_k'^3 X^3 - \frac{685}{144} \Sigma_k^4 \Sigma_k'^4 X^3 + \frac{7}{12} \Sigma_k^6 \Sigma_k''^2 X^3 + \frac{21}{8} \Sigma_k^4 \Sigma_k'^2 X^4 + \frac{211}{24} \Sigma_k^5 \Sigma_k'^3 X^4 \\
& + \frac{175}{16} \Sigma_k^6 \Sigma_k'^4 X^4 - \frac{7}{16} \Sigma_k^8 \Sigma_k''^2 X^4 + \frac{1}{8} \Sigma_k^4 X^5 - \frac{27}{4} \Sigma_k^6 \Sigma_k'^2 X^5 - \frac{185}{12} \Sigma_k^7 \Sigma_k'^3 X^5 - \frac{319}{24} \Sigma_k^8 \Sigma_k'^4 X^5 \\
& + \frac{1}{8} \Sigma_k^{10} \Sigma_k''^2 X^5 - \frac{3}{8} \Sigma_k^6 X^6 + \frac{15}{2} \Sigma_k^8 \Sigma_k'^2 X^6 + \frac{51}{4} \Sigma_k^9 \Sigma_k'^3 X^6 + \frac{49}{6} \Sigma_k^{10} \Sigma_k'^4 X^6 + \frac{3}{8} \Sigma_k^8 X^7 \\
& - 3 \Sigma_k^{10} \Sigma_k'^2 X^7 - 4 \Sigma_k^{11} \Sigma_k'^3 X^7 - 2 \Sigma_k^{12} \Sigma_k'^4 X^7 \Big], \\
\Delta \tilde{K}_{65} = & \int \frac{d^4 k}{(2\pi)^4} \Big[-\frac{1}{12} \Sigma_k'^3 X + \frac{1}{48} \Sigma_k' X^2 - \frac{1}{2} \Sigma_k \Sigma_k'^2 X^2 + \frac{1}{3} \Sigma_k^2 \Sigma_k'^3 X^2 - \frac{1}{16} \Sigma_k X^3 - \frac{1}{12} \Sigma_k^2 \Sigma_k' X^3 \\
& + \frac{7}{3} \Sigma_k^3 \Sigma_k'^2 X^3 - \frac{5}{12} \Sigma_k^4 \Sigma_k'^3 X^3 + \frac{29}{48} \Sigma_k^3 X^4 - \frac{19}{6} \Sigma_k^5 \Sigma_k'^2 X^4 + \frac{1}{6} \Sigma_k^6 \Sigma_k'^3 X^4 - \frac{1}{3} \Sigma_k^5 X^5 \\
& + \frac{4}{3} \Sigma_k^7 \Sigma_k'^2 X^5 \Big], \\
\Delta \tilde{K}_{66} = & \int \frac{d^4 k}{(2\pi)^4} \Big[\frac{1}{16} \Sigma_k'^2 X - \frac{3}{16} \Sigma_k^2 \Sigma_k'^2 X^2 - \frac{1}{8} \Sigma_k^2 X^3 + \frac{1}{8} \Sigma_k^4 \Sigma_k'^2 X^3 \Big], \\
\Delta \tilde{K}_{67} = & \int \frac{d^4 k}{(2\pi)^4} \Big[\frac{1}{72} \Sigma_k''^2 - \frac{1}{180} \Sigma_k \Sigma_k''' X + \frac{4}{27} \Sigma_k'^4 X - \frac{1}{18} \Sigma_k^2 \Sigma_k''^2 X + \frac{14}{45} \Sigma_k'^2 X^2 - \frac{1}{45} \Sigma_k \Sigma_k'' X^2 \\
& - \frac{11}{30} \Sigma_k \Sigma_k'^3 X^2 + \frac{1}{180} \Sigma_k^3 \Sigma_k''' X^2 - \frac{23}{27} \Sigma_k^2 \Sigma_k'^4 X^2 + \frac{5}{72} \Sigma_k^4 \Sigma_k''^2 X^2 + \frac{1}{180} X^3 + \frac{1}{45} \Sigma_k \Sigma_k' X^3 \\
& - \frac{31}{36} \Sigma_k^2 \Sigma_k'^2 X^3 + \frac{8}{5} \Sigma_k^3 \Sigma_k'^3 X^3 + \frac{46}{27} \Sigma_k^4 \Sigma_k'^4 X^3 - \frac{1}{36} \Sigma_k^6 \Sigma_k''^2 X^3 - \frac{7}{30} \Sigma_k^2 X^4 + \frac{3}{20} \Sigma_k^4 \Sigma_k'^2 X^4 \\
& - \frac{191}{90} \Sigma_k^5 \Sigma_k'^3 X^4 - \frac{13}{9} \Sigma_k^6 \Sigma_k'^4 X^4 + \frac{1}{3} \Sigma_k^6 \Sigma_k'^2 X^5 + \frac{8}{9} \Sigma_k^7 \Sigma_k'^3 X^5 + \frac{4}{9} \Sigma_k^8 \Sigma_k'^4 X^5 \Big], \\
\Delta \tilde{K}_{68} = & \int \frac{d^4 k}{(2\pi)^4} \Big[\frac{1}{36} \Sigma_k''^2 - \frac{1}{240} \Sigma_k \Sigma_k''' X + \frac{8}{27} \Sigma_k'^4 X - \frac{1}{9} \Sigma_k^2 \Sigma_k''^2 X + \frac{1}{40} \Sigma_k'^2 X^2 - \frac{1}{60} \Sigma_k \Sigma_k'' X^2 \\
& - \frac{83}{120} \Sigma_k \Sigma_k'^3 X^2 + \frac{1}{240} \Sigma_k^3 \Sigma_k''' X^2 - \frac{46}{27} \Sigma_k^2 \Sigma_k'^4 X^2 + \frac{5}{36} \Sigma_k^4 \Sigma_k''^2 X^2 + \frac{1}{240} X^3 + \frac{1}{60} \Sigma_k \Sigma_k' X^3 \\
& + \frac{41}{48} \Sigma_k^2 \Sigma_k'^2 X^3 + \frac{123}{40} \Sigma_k^3 \Sigma_k'^3 X^3 + \frac{92}{27} \Sigma_k^4 \Sigma_k'^4 X^3 - \frac{1}{18} \Sigma_k^6 \Sigma_k''^2 X^3 + \frac{1}{80} \Sigma_k^2 X^4 - \frac{103}{40} \Sigma_k^4 \Sigma_k'^2 X^4
\end{aligned}$$

$$-\frac{749}{180}\Sigma_k^5\Sigma_k'^3X^4 - \frac{26}{9}\Sigma_k^6\Sigma_k'^4X^4 - \frac{1}{4}\Sigma_k^4X^5 + \frac{5}{3}\Sigma_k^6\Sigma_k'^2X^5 + \frac{16}{9}\Sigma_k^7\Sigma_k'^3X^5 + \frac{8}{9}\Sigma_k^8\Sigma_k'^4X^5 \Big]. \quad (\text{A1})$$

Appendix B: The $\Delta\tilde{K}_i^W$ coefficients

$$\begin{aligned} \Delta\tilde{K}_1^W &= \int \frac{d^4k}{(2\pi)^4} \left[-\frac{1}{2}\Sigma_k^5X^5 \right], \\ \Delta\tilde{K}_2^W &= \int \frac{d^4k}{(2\pi)^4} \left[-\frac{1}{8}\Sigma_k^3X^4 + \frac{1}{8}\Sigma_k^5X^5 \right], \\ \Delta\tilde{K}_3^W &= \int \frac{d^4k}{(2\pi)^4} \left[-\frac{1}{8}\Sigma_k^3X^4 \right], \\ \Delta\tilde{K}_4^W &= \int \frac{d^4k}{(2\pi)^4} \left[-\frac{1}{8}\Sigma_k^3X^4 \right], \\ \Delta\tilde{K}_5^W &= \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{32}\Sigma_kX^3 \right], \\ \Delta\tilde{K}_6^W &= \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{32}\Sigma_k^3X^4 \right], \\ \Delta\tilde{K}_7^W &= \int \frac{d^4k}{(2\pi)^4} \left[-\frac{1}{64}\Sigma_kX^3 + \frac{1}{64}\Sigma_k^3X^4 \right], \\ \Delta\tilde{K}_8^W &= \int \frac{d^4k}{(2\pi)^4} \left[-\frac{1}{32}\Sigma_k'^2X^2 - \frac{11}{24}\Sigma_k\Sigma_k'^3X^2 + \frac{19}{192}X^3 + \frac{1}{3}\Sigma_k\Sigma_k'X^3 + \frac{5}{24}\Sigma_k^2\Sigma_k'^2X^3 + \frac{13}{6}\Sigma_k^3\Sigma_k'^3X^3 \right. \\ &\quad \left. - \frac{25}{192}\Sigma_k^2X^4 + \frac{59}{96}\Sigma_k^4\Sigma_k'^2X^4 - \frac{95}{24}\Sigma_k^5\Sigma_k'^3X^4 - \frac{1}{24}\Sigma_k^4X^5 - \frac{43}{24}\Sigma_k^6\Sigma_k'^2X^5 + \frac{47}{12}\Sigma_k^7\Sigma_k'^3X^5 \right. \\ &\quad \left. - \frac{3}{4}\Sigma_k^6X^6 - \frac{7}{3}\Sigma_k^9\Sigma_k'^3X^6 - \frac{1}{6}\Sigma_k^8X^7 + \Sigma_k^{10}\Sigma_k'^2X^7 + \frac{2}{3}\Sigma_k^{11}\Sigma_k'^3X^7 \right], \\ \Delta\tilde{K}_9^W &= \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{32}\Sigma_k'^2X^2 + \frac{5}{8}\Sigma_k\Sigma_k'^3X^2 - \frac{9}{64}X^3 - \frac{1}{2}\Sigma_k\Sigma_k'X^3 - \frac{7}{8}\Sigma_k^2\Sigma_k'^2X^3 - 3\Sigma_k^3\Sigma_k'^3X^3 \right. \\ &\quad \left. + \frac{11}{64}\Sigma_k^2X^4 + \frac{181}{96}\Sigma_k^4\Sigma_k'^2X^4 + \frac{45}{8}\Sigma_k^5\Sigma_k'^3X^4 + \frac{3}{8}\Sigma_k^4X^5 - \frac{29}{24}\Sigma_k^6\Sigma_k'^2X^5 - \frac{23}{4}\Sigma_k^7\Sigma_k'^3X^5 \right. \\ &\quad \left. + \frac{13}{12}\Sigma_k^6X^6 + \frac{5}{3}\Sigma_k^8\Sigma_k'^2X^6 + \frac{7}{2}\Sigma_k^9\Sigma_k'^3X^6 + \frac{1}{4}\Sigma_k^8X^7 - \frac{3}{2}\Sigma_k^{10}\Sigma_k'^2X^7 - \Sigma_k^{11}\Sigma_k'^3X^7 \right], \\ \Delta\tilde{K}_{10}^W &= \int \frac{d^4k}{(2\pi)^4} \left[\frac{7}{96}\Sigma_k'^2X^2 + \frac{5}{24}\Sigma_k\Sigma_k'^3X^2 - \frac{19}{192}X^3 - \frac{1}{3}\Sigma_k\Sigma_k'X^3 + \frac{1}{3}\Sigma_k^2\Sigma_k'^2X^3 + \frac{5}{6}\Sigma_k^3\Sigma_k'^3X^3 \right. \\ &\quad \left. + \frac{17}{192}\Sigma_k^2X^4 - \frac{635}{96}\Sigma_k^4\Sigma_k'^2X^4 - \frac{175}{24}\Sigma_k^5\Sigma_k'^3X^4 - \frac{5}{8}\Sigma_k^4X^5 + \frac{143}{8}\Sigma_k^6\Sigma_k'^2X^5 + \frac{175}{12}\Sigma_k^7\Sigma_k'^3X^5 \right. \\ &\quad \left. + \frac{7}{3}\Sigma_k^6X^6 - \frac{50}{3}\Sigma_k^8\Sigma_k'^2X^6 - \frac{35}{3}\Sigma_k^9\Sigma_k'^3X^6 - \frac{5}{6}\Sigma_k^8X^7 + 5\Sigma_k^{10}\Sigma_k'^2X^7 + \frac{10}{3}\Sigma_k^{11}\Sigma_k'^3X^7 \right], \\ \Delta\tilde{K}_{11}^W &= \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{24}\Sigma_k'^2X^2 + \frac{1}{3}\Sigma_k\Sigma_k'^3X^2 - \frac{13}{192}X^3 - \frac{5}{24}\Sigma_k\Sigma_k'X^3 - \frac{19}{48}\Sigma_k^2\Sigma_k'^2X^3 - \frac{3}{2}\Sigma_k^3\Sigma_k'^3X^3 \right. \\ &\quad \left. - \frac{7}{192}\Sigma_k^2X^4 + \frac{17}{48}\Sigma_k^4\Sigma_k'^2X^4 + \frac{5}{2}\Sigma_k^5\Sigma_k'^3X^4 + \frac{25}{48}\Sigma_k^4X^5 + \frac{1}{2}\Sigma_k^6\Sigma_k'^2X^5 - \frac{13}{6}\Sigma_k^7\Sigma_k'^3X^5 \right. \\ &\quad \left. + \frac{1}{24}\Sigma_k^6X^6 + \frac{7}{6}\Sigma_k^9\Sigma_k'^3X^6 + \frac{1}{12}\Sigma_k^8X^7 - \frac{1}{2}\Sigma_k^{10}\Sigma_k'^2X^7 - \frac{1}{3}\Sigma_k^{11}\Sigma_k'^3X^7 \right], \\ \Delta\tilde{K}_{12}^W &= \int \frac{d^4k}{(2\pi)^4} \left[-\frac{1}{24}\Sigma_k'^2X^2 - \frac{11}{24}\Sigma_k\Sigma_k'^3X^2 + \frac{1}{12}X^3 + \frac{1}{3}\Sigma_k\Sigma_k'X^3 + \frac{5}{16}\Sigma_k^2\Sigma_k'^2X^3 + \frac{13}{6}\Sigma_k^3\Sigma_k'^3X^3 \right. \\ &\quad \left. - \frac{7}{24}\Sigma_k^2X^4 + \frac{7}{16}\Sigma_k^4\Sigma_k'^2X^4 - \frac{95}{24}\Sigma_k^5\Sigma_k'^3X^4 - \frac{41}{24}\Sigma_k^6\Sigma_k'^2X^5 + \frac{47}{12}\Sigma_k^7\Sigma_k'^3X^5 - \frac{1}{2}\Sigma_k^6X^6 \right] \end{aligned}$$

$$\begin{aligned}
& -\frac{7}{3}\Sigma_k^9\Sigma_k'^3X^6 - \frac{1}{6}\Sigma_k^8X^7 + \Sigma_k^{10}\Sigma_k'^2X^7 + \frac{2}{3}\Sigma_k^{11}\Sigma_k'^3X^7 \Big], \\
\Delta\tilde{K}_{13}^W &= \int \frac{d^4k}{(2\pi)^4} \left[-\frac{1}{8}\Sigma_k^2X^4 + \frac{1}{8}\Sigma_k^4X^5 \right], \\
\Delta\tilde{K}_{14}^W &= \int \frac{d^4k}{(2\pi)^4} \left[\frac{5}{96}\Sigma_k'^2X^2 + \frac{2}{3}\Sigma_k\Sigma_k'^3X^2 - \frac{23}{192}X^3 - \frac{5}{12}\Sigma_k\Sigma_k'X^3 - \frac{31}{48}\Sigma_k^2\Sigma_k'^2X^3 - 3\Sigma_k^3\Sigma_k'^3X^3 \right. \\
& + \frac{1}{192}\Sigma_k^2X^4 + \frac{49}{96}\Sigma_k^4\Sigma_k'^2X^4 + 5\Sigma_k^5\Sigma_k'^3X^4 + \frac{13}{24}\Sigma_k^4X^5 + \frac{13}{12}\Sigma_k^6\Sigma_k'^2X^5 - \frac{13}{3}\Sigma_k^7\Sigma_k'^3X^5 \\
& + \frac{1}{2}\Sigma_k^6X^6 + \frac{7}{3}\Sigma_k^9\Sigma_k'^3X^6 + \frac{1}{6}\Sigma_k^8X^7 - \Sigma_k^{10}\Sigma_k'^2X^7 - \left. \frac{2}{3}\Sigma_k^{11}\Sigma_k'^3X^7 \right], \\
\Delta\tilde{K}_{15}^W &= \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{96}\Sigma_k'^2X^2 + \frac{1}{48}\Sigma_k\Sigma_k'^3X^2 + \frac{1}{96}X^3 + \frac{1}{24}\Sigma_k\Sigma_k'X^3 + \frac{19}{96}\Sigma_k^2\Sigma_k'^2X^3 - \frac{7}{12}\Sigma_k^3\Sigma_k'^3X^3 \right. \\
& + \frac{5}{48}\Sigma_k^2X^4 - \frac{3}{16}\Sigma_k^4\Sigma_k'^2X^4 + \frac{125}{48}\Sigma_k^5\Sigma_k'^3X^4 - \frac{19}{48}\Sigma_k^4X^5 - \frac{89}{48}\Sigma_k^6\Sigma_k'^2X^5 - \frac{109}{24}\Sigma_k^7\Sigma_k'^3X^5 \\
& - \frac{1}{12}\Sigma_k^6X^6 + \frac{10}{3}\Sigma_k^8\Sigma_k'^2X^6 + \frac{7}{2}\Sigma_k^9\Sigma_k'^3X^6 + \frac{1}{4}\Sigma_k^8X^7 - \frac{3}{2}\Sigma_k^{10}\Sigma_k'^2X^7 - \left. \Sigma_k^{11}\Sigma_k'^3X^7 \right], \\
\Delta\tilde{K}_{16}^W &= \int \frac{d^4k}{(2\pi)^4} \left[-\frac{1}{96}\Sigma_k'^2X^2 - \frac{19}{24}\Sigma_k\Sigma_k'^3X^2 + \frac{23}{192}X^3 + \frac{5}{12}\Sigma_k\Sigma_k'X^3 + \frac{19}{24}\Sigma_k^2\Sigma_k'^2X^3 + \frac{9}{2}\Sigma_k^3\Sigma_k'^3X^3 \right. \\
& - \frac{11}{64}\Sigma_k^2X^4 - \frac{319}{96}\Sigma_k^4\Sigma_k'^2X^4 - \frac{85}{8}\Sigma_k^5\Sigma_k'^3X^4 - \frac{7}{12}\Sigma_k^4X^5 + \frac{55}{8}\Sigma_k^6\Sigma_k'^2X^5 + \frac{163}{12}\Sigma_k^7\Sigma_k'^3X^5 \\
& + \frac{1}{12}\Sigma_k^6X^6 - \frac{25}{3}\Sigma_k^8\Sigma_k'^2X^6 - \frac{28}{3}\Sigma_k^9\Sigma_k'^3X^6 - \frac{2}{3}\Sigma_k^8X^7 + 4\Sigma_k^{10}\Sigma_k'^2X^7 + \left. \frac{8}{3}\Sigma_k^{11}\Sigma_k'^3X^7 \right], \\
\Delta\tilde{K}_{17}^W &= \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{24}\Sigma_k'^2X^2 - \frac{17}{24}\Sigma_k\Sigma_k'^3X^2 + \frac{1}{12}X^3 + \frac{1}{3}\Sigma_k\Sigma_k'X^3 - \frac{1}{16}\Sigma_k^2\Sigma_k'^2X^3 + \frac{31}{6}\Sigma_k^3\Sigma_k'^3X^3 \right. \\
& - \frac{1}{6}\Sigma_k^2X^4 - \frac{35}{16}\Sigma_k^4\Sigma_k'^2X^4 - \frac{365}{24}\Sigma_k^5\Sigma_k'^3X^4 + \frac{245}{24}\Sigma_k^6\Sigma_k'^2X^5 + \frac{269}{12}\Sigma_k^7\Sigma_k'^3X^5 + \frac{1}{4}\Sigma_k^6X^6 \\
& - 15\Sigma_k^8\Sigma_k'^2X^6 - \frac{49}{3}\Sigma_k^9\Sigma_k'^3X^6 - \frac{7}{6}\Sigma_k^8X^7 + 7\Sigma_k^{10}\Sigma_k'^2X^7 + \left. \frac{14}{3}\Sigma_k^{11}\Sigma_k'^3X^7 \right], \\
\Delta\tilde{K}_{18}^W &= \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{12}\Sigma_k'^2X^2 + \frac{2}{3}\Sigma_k\Sigma_k'^3X^2 - \frac{13}{96}X^3 - \frac{5}{12}\Sigma_k\Sigma_k'X^3 - \frac{19}{24}\Sigma_k^2\Sigma_k'^2X^3 - 3\Sigma_k^3\Sigma_k'^3X^3 \right. \\
& - \frac{1}{32}\Sigma_k^2X^4 + \frac{17}{24}\Sigma_k^4\Sigma_k'^2X^4 + 5\Sigma_k^5\Sigma_k'^3X^4 + \frac{7}{12}\Sigma_k^4X^5 + \Sigma_k^6\Sigma_k'^2X^5 - \frac{13}{3}\Sigma_k^7\Sigma_k'^3X^5 \\
& + \frac{1}{2}\Sigma_k^6X^6 + \frac{7}{3}\Sigma_k^9\Sigma_k'^3X^6 + \frac{1}{6}\Sigma_k^8X^7 - \Sigma_k^{10}\Sigma_k'^2X^7 - \left. \frac{2}{3}\Sigma_k^{11}\Sigma_k'^3X^7 \right], \\
\Delta\tilde{K}_{19}^W &= \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{96}\Sigma_k'^2X^2 - \frac{1}{4}\Sigma_k\Sigma_k'^3X^2 + \frac{1}{64}X^3 + \frac{11}{16}\Sigma_k^2\Sigma_k'^2X^3 + 3\Sigma_k^3\Sigma_k'^3X^3 - \frac{3}{64}\Sigma_k^2X^4 \right. \\
& - \frac{595}{96}\Sigma_k^4\Sigma_k'^2X^4 - \frac{45}{4}\Sigma_k^5\Sigma_k'^3X^4 - \frac{1}{4}\Sigma_k^4X^5 + \frac{97}{6}\Sigma_k^6\Sigma_k'^2X^5 + \frac{37}{2}\Sigma_k^7\Sigma_k'^3X^5 + \frac{7}{6}\Sigma_k^6X^6 \\
& - \frac{50}{3}\Sigma_k^8\Sigma_k'^2X^6 - 14\Sigma_k^9\Sigma_k'^3X^6 - \Sigma_k^8X^7 + 6\Sigma_k^{10}\Sigma_k'^2X^7 + \left. 4\Sigma_k^{11}\Sigma_k'^3X^7 \right], \\
\Delta\tilde{K}_{20}^W &= \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{24}\Sigma_k'^2X^2 + \frac{5}{12}\Sigma_k\Sigma_k'^3X^2 - \frac{1}{24}X^3 - \frac{1}{6}\Sigma_k\Sigma_k'X^3 - \frac{7}{8}\Sigma_k^2\Sigma_k'^2X^3 - \frac{5}{3}\Sigma_k^3\Sigma_k'^3X^3 \right. \\
& + \frac{1}{8}\Sigma_k^2X^4 + \frac{9}{4}\Sigma_k^4\Sigma_k'^2X^4 + \frac{25}{12}\Sigma_k^5\Sigma_k'^3X^4 + \frac{1}{4}\Sigma_k^4X^5 - \frac{17}{12}\Sigma_k^6\Sigma_k'^2X^5 - \frac{5}{6}\Sigma_k^7\Sigma_k'^3X^5 \Big], \\
\Delta\tilde{K}_{21}^W &= \int \frac{d^4k}{(2\pi)^4} \left[-\frac{1}{4}\Sigma_k\Sigma_k'^3X^2 + \frac{19}{24}\Sigma_k^2\Sigma_k'^2X^3 + 3\Sigma_k^3\Sigma_k'^3X^3 + \frac{1}{24}\Sigma_k^2X^4 - \frac{51}{8}\Sigma_k^4\Sigma_k'^2X^4 - \frac{45}{4}\Sigma_k^5\Sigma_k'^3X^4 \right. \\
& - \frac{1}{3}\Sigma_k^4X^5 + \frac{65}{4}\Sigma_k^6\Sigma_k'^2X^5 + \frac{37}{2}\Sigma_k^7\Sigma_k'^3X^5 + \frac{7}{6}\Sigma_k^6X^6 - \frac{50}{3}\Sigma_k^8\Sigma_k'^2X^6 - 14\Sigma_k^9\Sigma_k'^3X^6 \\
& \left. - \Sigma_k^8X^7 + 6\Sigma_k^{10}\Sigma_k'^2X^7 + 4\Sigma_k^{11}\Sigma_k'^3X^7 \right],
\end{aligned}$$

$$\begin{aligned}
\Delta \tilde{K}_{22}^W &= \int \frac{d^4 k}{(2\pi)^4} \left[\frac{1}{16} \Sigma_k'^2 X^2 + \frac{1}{16} \Sigma_k \Sigma_k'^3 X^2 - \frac{13}{32} \Sigma_k^2 \Sigma_k'^2 X^3 - \frac{1}{4} \Sigma_k^3 \Sigma_k'^3 X^3 - \frac{1}{16} \Sigma_k^2 X^4 + \frac{21}{32} \Sigma_k^4 \Sigma_k'^2 X^4 \right. \\
&\quad \left. + \frac{5}{16} \Sigma_k^5 \Sigma_k'^3 X^4 + \frac{1}{16} \Sigma_k^4 X^5 - \frac{5}{16} \Sigma_k^6 \Sigma_k'^2 X^5 - \frac{1}{8} \Sigma_k^7 \Sigma_k'^3 X^5 \right], \\
\Delta \tilde{K}_{23}^W &= \int \frac{d^4 k}{(2\pi)^4} \left[-\frac{1}{8} \Sigma_k \Sigma_k'^3 X^2 + \frac{9}{16} \Sigma_k^2 \Sigma_k'^2 X^3 + \frac{3}{2} \Sigma_k^3 \Sigma_k'^3 X^3 - \frac{63}{16} \Sigma_k^4 \Sigma_k'^2 X^4 - \frac{45}{8} \Sigma_k^5 \Sigma_k'^3 X^4 - \frac{1}{4} \Sigma_k^4 X^5 \right. \\
&\quad \left. + \frac{73}{8} \Sigma_k^6 \Sigma_k'^2 X^5 + \frac{37}{4} \Sigma_k^7 \Sigma_k'^3 X^5 + \frac{11}{16} \Sigma_k^6 X^6 - \frac{35}{4} \Sigma_k^8 \Sigma_k'^2 X^6 - 7 \Sigma_k^9 \Sigma_k'^3 X^6 - \frac{1}{2} \Sigma_k^8 X^7 \right. \\
&\quad \left. + 3 \Sigma_k^{10} \Sigma_k'^2 X^7 + 2 \Sigma_k^{11} \Sigma_k'^3 X^7 \right].
\end{aligned} \tag{B1}$$

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